Mechanics of Systems – Test 1

Monday 9th December 2024 - 1h30 (10h15-11h45)

Personal 1,5-A4 page formula sheet + table of joints and calculators are authorised.

Marking scale for the 2 independent parts: part A : 15 marks ; part B : 5 marks.

Part A : STATIC ANALYSIS of a harbour crane (≈15 marks)

For most harbour cranes (Figures 1.1 and 1.2), it is interesting to minimise the phases when loads are lifted up. Once a load has been removed from a ship hold, the objective is therefore to translate it horizontally in order to reduce energy consumption.

Figure 1.1 : Harbour cranes

OBJECTIVE OF THE ANALYSIS

Determine, at equilibrium, the forces exerted by the hydraulic actuators for a given load to be moved.

MODEL

The crane under consideration has a load carrying capacity of 50 tons and comprises (Figure 1.3) :

- a turret assimilated to the **ground 0**,
- a **boom 1** connected to the ground 0 via a revolute joint (pin connection) of axis $\left(E,\vec{y}\right)$ and motion parameter **1/0** $\alpha=\left(\vec{x}_0,\vec{x}_1\right)$

  and connected to the arm **2** via a revolute joint of axis $\left(B,\vec{y}\right)$ and motion parameter **2/1** $\beta=\left(\vec{x}_1,\vec{x}_2\right)$.

  The load is applied at point C at the extremity of **arm 2**.

- a **rod 3** that supports arm **2** and is connected to the ground **0** via a revolute joint of axis $\left(D,\vec{y}\right)$ and motion parameter **3/0** $\delta=\left(\vec{x}_0,\vec{x}_3\right)$

  and connected to the arm **2** via a revolute joint of axis $\left(A,\vec{y}\right)$. No parameter is introduced for this joint.

The motion of arm 2 is controlled by a hydraulic jack comprising:

- a **cylinder 5** connected to the ground **0** via a revolute joint of axis $\left(F,\vec{y}\right)$ and motion parameter **5/0** $\varphi=\left(\vec{x}_0,\vec{x}_5\right)$ and connected to **piston 6** via a prismatic joint (slider) of direction $\vec{x}_5$ and translational motion parameter **6/5** λ, such that $\overrightarrow{FH}=\lambda\vec{x}_5$.
- the piston **6** of the jack is connected to boom **1** via a revolute joint of axis $\left(H,\vec{y}\right)$. No parameter is introduced for this joint.

HYPOTHESES

- The problem is **planar.**
- All the joints are perfect.
- The weights of the various parts of the crane are neglected compared with the load at point C.
- The whole system is in equilibrium.
- The downward vertical axis is denoted $\vec{z}_0$.

DATA

- The force F_c generated by the load carried by the arm 2 is known and the corresponding wrench at point C reads: $\left\{F_{load/2}\right\}=\begin{Bmatrix}F_c\vec{z}_0\\ \vec{0}\end{Bmatrix}_C$
- The overall fluid force $\left\|\vec{F}_{Fluid/6}\right\|$ in the direction of the piston rod of jack **6** is the scalar unknown that has to be determined.

NOTATIONS

The mechanical actions of solid i $\left(i\in\{1,2,...,6\}\right)$ onto solid j $\left(j\in\{1,2,...,6\},\ j\neq i\right)$ will be represented by a wrench at point P $\left(A,B,....,H\right)$, and expressed in the coordinate system R$\left(R_1,R_2,...,R_5\right)$ as :

$$\left\{\mathrm{F}_{i/j}\right\}:\begin{Bmatrix}\vec{F}_{i/j}\\ \vec{\mathrm{M}}_{i/j}(P)\end{Bmatrix}_P=\begin{Bmatrix}\begin{pmatrix}X_{ij}\\ Y_{ij}\\ Z_{ij}\end{pmatrix}_R & \begin{pmatrix}L_{ij}\\ M_{ij}\\ N_{ij}\end{pmatrix}_R\end{Bmatrix}_P$$

QUESTIONS

11 - Determine the total number of static unknowns for the complete mechanism.

12 – Isolate rod **3**.

- Develop the equilibrium equations at point D.
- Deduce the particular form of the external force wrenches exerted on the rod.

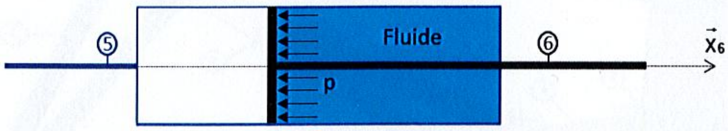
13 – Isolate arm **2**.

- Develop the static equilibrium equations at point A projected in the coordinate system R_0. Do not try to solve.

In the same way, the static analysis of the boom **1** and that of the system **5U6** make it possible to determine the various wrenches of mechanical actions. In particular, the following wrench is now supposed to be known (in terms of geometrical data and force F_c):

$$\{F_{1/6}\}:\left\{\begin{pmatrix} X_{16} \\ - \\ 0 \end{pmatrix}_{R_5} \begin{pmatrix} - \\ 0 \\ - \end{pmatrix}_{R_5}\right\}_H$$

In order to determine the force $\vec{F}_{Fluid/6}$, one considers the isolated piston **6** of the actuator. The pressure p applied on the circular surface of radius r of piston **6** is considered uniform.



14 – Determine the force wrench generated by the mechanical action of the fluid on the piston **6** $\{F_{Fluid/6}\}$ at point F in terms of radius r and pressure p, knowing that the active surface of the piston is perpendicular to the piston rod **6.** Justify your reasoning.

15 - Equilibrium of the piston **6**.
- Develop the static equilibrium equations at point H.
- Calculate the oil pressure in the jack in terms of the force component X_{16} and radius r.

16 - Determine the force wrench $\{F_{6/5}\}$. Conclude.

Part B : KINEMATICS (≈5 marks)

The following study is based on the harbour crane analysed in part A (Statics). For the system description, please refer to the previous part.

The objectives are to determine constraint equations, velocity and acceleration vectors at points H, A and B of the arm **2**.

Questions

21 – Draw the graph of links of the mechanism.

22 – Draw the associated change of basis diagrams.

23 – Give and develop the constraint equation(s) associated with the closure by the non-parametered joint at H between 6 and 1, in terms of certain parameters and geometric data.

24 – What is the nature of motion 5/6 ? Calculate the velocity and acceleration vectors at point H in its motion **6** with respect to **5**. Express the result in terms of the translational parameter λ.

25 – What is the trajectory of H with respect to **0** ? Calculate the velocity and acceleration vectors of point H in its motion with respect to **0**.

26 – Express the instant angular velocity vector for the motion of **3** with respect to **0**.

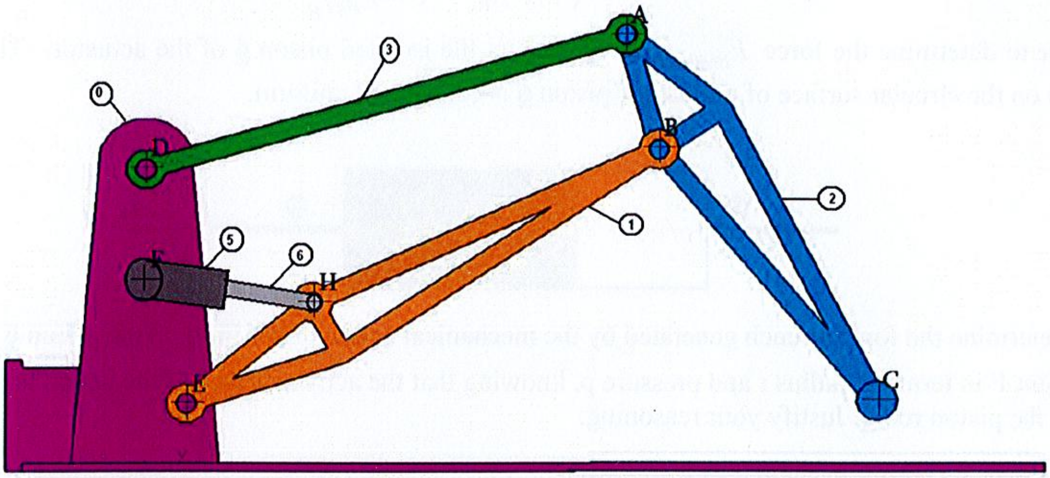


Figure 1.2 : Harbour crane model

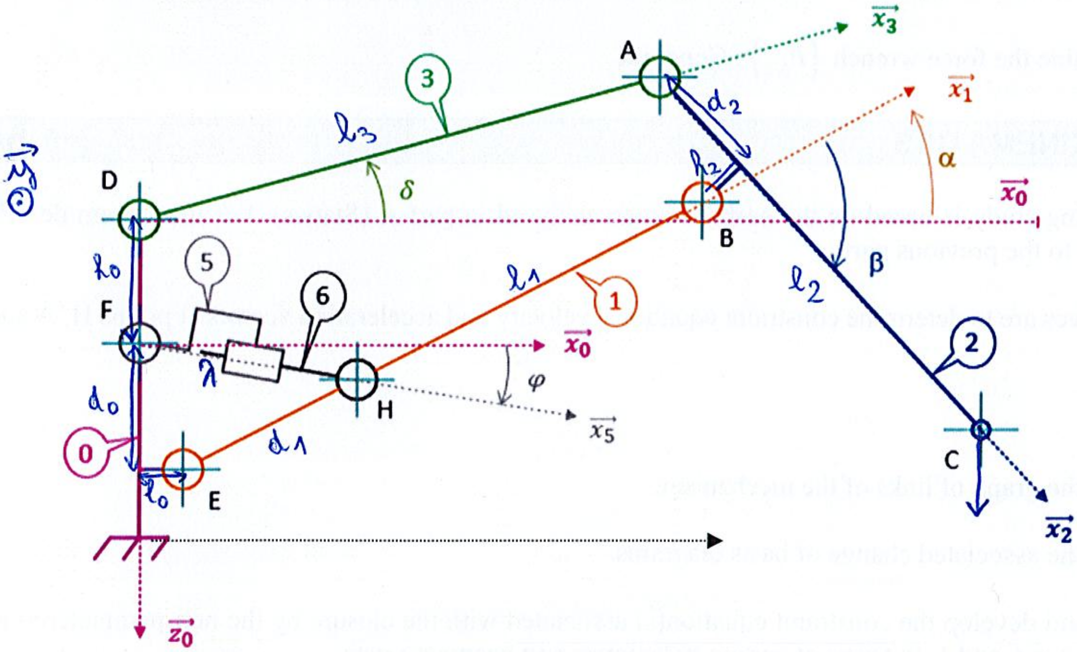


Figure 1.3 : Harbour crane model – coordinate systems and solid labels.

Geometrical data

$$\overrightarrow{FE} = l_0\overrightarrow{z_0} + d_0\overrightarrow{x_0}$$
$$\overrightarrow{FD} = -h_0\overrightarrow{z_0}$$
$$\overrightarrow{DA} = l_3\overrightarrow{x_3}$$
$$\overrightarrow{AB} = d_2\overrightarrow{x_2} + h_2\overrightarrow{z_2}$$
$$\overrightarrow{AC} = l_2\overrightarrow{x_2}$$
$$\overrightarrow{EH} = d_1\overrightarrow{x_1}$$
$$\overrightarrow{HB} = l_1\overrightarrow{x_1}$$