INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

PC Second Year **Year 2015/2016**

Physics : Test n°4

Thursday May 12th, 2016 **Duration : 1h30**

Approximate marking scheme: Exercise 1: 10 points ; Exercise 2: 10 points

Exercise I: Interpretation of the shimmering colours of peacock feathers

« The coloured decoration of Hindu temples, symbol of vanity and luxury in Europe, graphical representation of immortality and of the sun or synonymous with dignity in the East, the intriguing blue bird, surrounded by mystery and superstitions, never left human beings insensitive. »

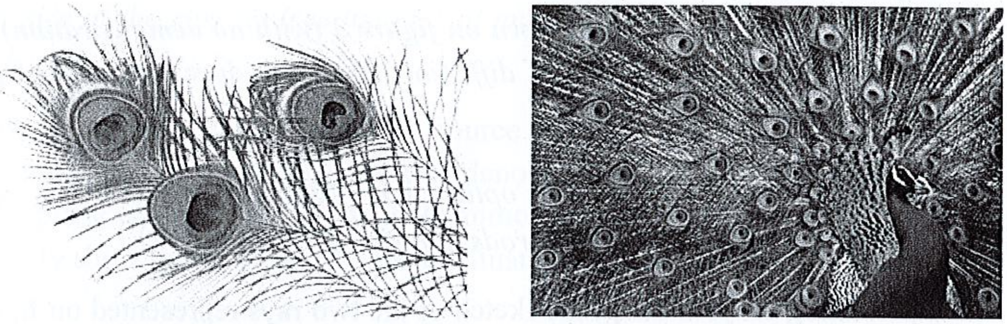

Figure 1

Peacock feathers contain melanin rods (melanin is a dark brown pigment which gives human skin its colour). These melanin rods are opaque and located within a keratin matrix (transparent medium of index n=1.5). Each rod constitutes a reflecting obstacle that diffracts light.

1) *We consider here rods regularly distributed within the keratin located at nodes inter-spaced by a distance* ***a*** *as represented in figure 2 below. Each node, denoted (j,k), is defined by the 2 indices j and k. The nodes located on the same horizontal line parallel to the Oz axis have the same value for j and a value for k comprised between 0 and N_z -1. In the same way, the nodes located on the same vertical line, parallel to the Oy axis, have the same value for k and a value for j comprised between 0 and N_y -1. Point O is located at the interface air/keratin. N_y is the number of rods in the y direction and N_z is the number of rods in the z direction.*

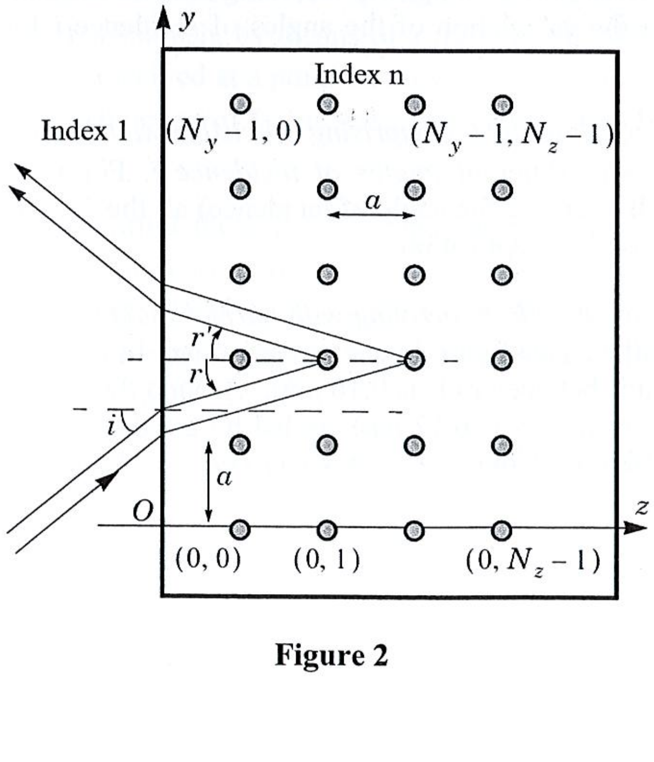

Figure 2

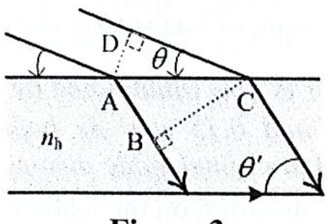

Figure 3

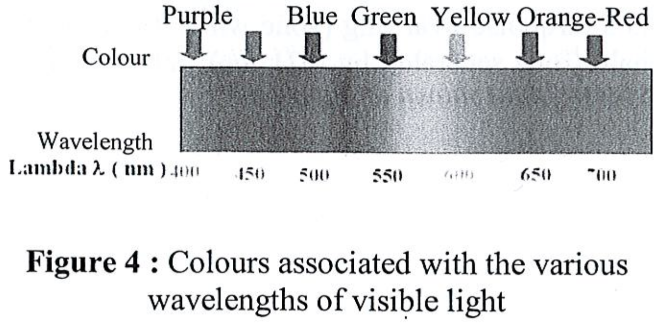

Figure 4 : Colours associated with the various wavelengths of visible light

Working assumptions and instructions:

We will assume that the rods are of length L in the Ox direction (perpendicular to the plane of Figure 2) with L>>a and L much larger than the wavelength.

- *The feather is illuminated by a parallel beam of angle of incidence i (see figure 2). The light is refracted at the interface with an angle of refraction r.* ***We will consider here only the light reflected by each rod in the direction r' = – r.*** *This direction corresponds to the one given by Snell Descartes reflection law. On Figure 2 two rays are represented which are reflected by two adjacent rods, on the same coordinate y.*

- ***We will accept the following result shown on figure 3 (with no demonstration):*** *the refraction at an interface between two media of different index introduces no optical path difference between the rays, hence nAB = CD.*

1.1- *Let δ_z be the difference in optical path between the two waves reflected by two adjacent rods of the same coordinate y: rods at nodes (j,k) and (j,k+1)*

Reproducing on your script the sketch of the two rays represented on figure 2, highlight clearly the difference in optical path and demonstrate that $\delta_z = 2na\sqrt{1-\frac{\sin^2 i}{n^2}}$. Deduce the phase shift, φ_z, between the wave reflected by the rod (j,k) and the wave reflected by the rod (j,k+1).

1.2- *Similarly, δ_y is the optical path difference between the two waves reflected by two adjacent rods with the same coordinate z, corresponding to nodes (j,k) and (j+1,k).* Starting from a sketch of the two rays reflected by these two rods, determine the optical path difference δ_y between these two waves. Deduce the phase shift φ_y between the waves reflected by these two rods.

2) *The feather is illuminated by a parallel beam of monochromatic light of wavelength λ_v in vacuum.* From the previous results, give the relation enabling the calculation of the angles of incidence *i* for which a maximum of intensity is obtained.

3) *The feather is now illuminated by a parallel beam of white light, comprising all visible wavelengths between 0.4 and 0.75 µm. As before, we will consider different angles of incidence i.* For what minimum value of *inter-node distance a* can we see (by varying the angle of incidence) all the colours of the visible spectrum on the feather? Perform the numerical application.

4) *The feather is now illuminated by a parallel beam of white light arriving with normal incidence i = 0.* On observing the colored pattern of the peacock feathers (see figure 1) one can see a dark blue centre (associated with a zone in the feather where the distance between rods is 0.16 *µm*). Around this centre is a turquoise oval ring (zone where the distance between rods is 0.17 *µm*) circled by a red-brownish halo (rods separated by 0,21 *µm*). Interpret these different colours. *For this you can use the visible spectra data shown on figure 4.*

Exercise II: Fresnel's mirror Interferometer

In some solar plants which use Fresnel reflecting concentrators, flat mirrors pivot and track the sun to concentrate its rays onto tubes. The solar energy is absorbed and converted into heat, up to 500°C, which is transferred to the heat-transfer fluid circulating through the tubes. The water vapour thus produced drives a turbine and produces electricity.

Here we will study a simplified version comprising two perfectly reflecting mirrors (coefficient of reflection equal to -1). The two identical mirrors, (M_1) and (M_2), are adjoined along an axis Δ forming a shallow angle α (we can use small angle approximations). The combination is located within air with an index taken as equal to 1. A monochromatic point source S, of wavelength λ in air, illuminates simultaneously the two mirrors. It is placed at a distance R from the axis Δ, R being considered large compared to the size of the mirrors (see figure 1 of annex 1 which must be completed and handed in with your script).

1) Show, tracing the outermost rays from the source S which illuminate the mirrors, that the light appears to come from two secondary sources, denoted S_1 and S_2, with S_1 (symmetric with S with respect to the plane of mirror M_1) and S_2 (symmetric with S with respect to the plane of mirror M_2). Graphically show that S, S_1 and S_2 are all situated on a circle of centre J (situated on axis Δ).

2) Draw the interference field associated with the two waves emanating from S_1 and S_2. Show that the angle γ between sources S_1 and S_2 as viewed from point J (i.e. angle S_1JS_2) is equal to 2α.

3) *A Screen (E) is placed perpendicularly to the perpendicular bisector of segment (S_1S_2) to observe the zone of interference. Let D denote the distance JO and x the direction perpendicular to (Δ) in the plane of the screen. The origin O of the axis (Ox) on the screen is located at the intersection between the perpendicular bisector of segment (S_1S_2) and the plane of (E).*

a) Sketch the setup clearly identifying all the key parameters and the zone of interference on the screen.

b) Describe the pattern of fringes which are observed on the screen and give the literal expression of the interfringe distance, *i*, as a function of λ, D, R and α.

c) Calculate *i* for $R = 0.1$ m, $D = 1.0$ m, $\lambda = 0.5$ µm and $\alpha = 9''$.

d) Establish, by means of a rigorous and detailed demonstration, the expression of the intensity I received at a point on the screen located at a distance x from point O. Express this intensity as a function of I_0 (the wave intensity emitted by the source S), x and *i*.

4) Determine the width L of the interference field and the maximum path difference at the edges of the field of observation.

Bonus: Under what condition are the fringes observed at the edges of the observation field if the source emits trains of waves of coherence length $L_c = c\tau$ where τ is the duration of the source's emission?

ANNEX 1 (to be completed and handed in with your script)

Family Name: **First Name:** **Group:**

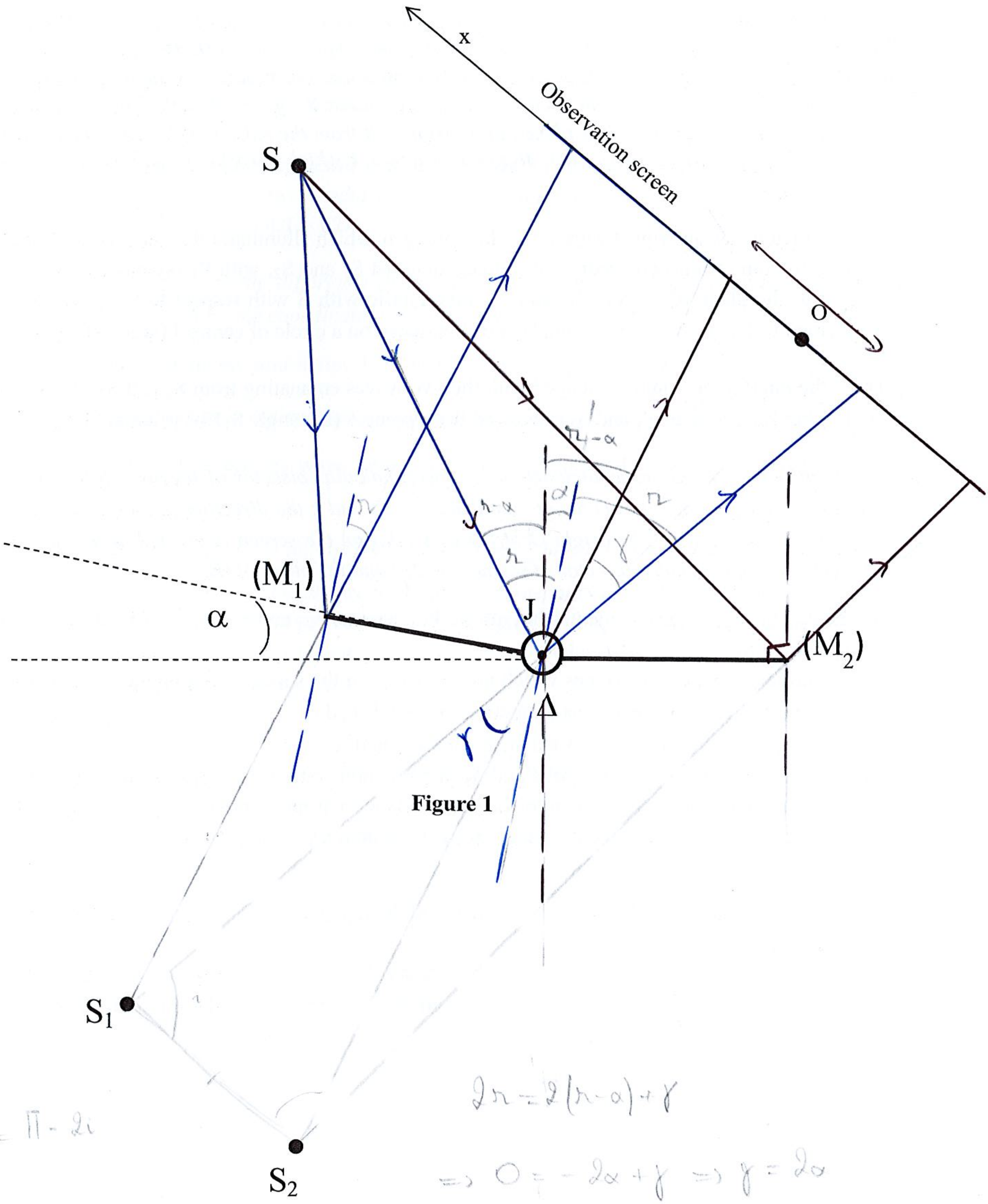

Figure 1