Long Test - Semester 2

Wednesday June 14th 2017, duration: 3hours

Instructions :

*Your results, as well as your **ability to justify them clearly** and to **analyze** and **comment on them** in a critical manner will be assessed. It is also recalled that you must pay attention to **spelling** and to the **presentation of your manuscript**. Calculator and formula sheet are allowed. **Any illogical result or dimension which doesn't make sense, given without comment will be penalized**.*

Documents permitted: 2 handwritten sheets altogether 4 pages, handwritten on both sides (no photocopies allowed)
College-type, non-programmable calculator only permitted.

Tentative marking scheme: Problem I: 10 points, Problem II: 10 points.
The subject is constituted of 2 completely independent problems.

PROBLEM I: STUDY OF A RECTANGULAR WAVEGUIDE

A waveguide is a rigid or flexible system aiming at guiding electromagnetic or acoustic waves, by use of the reflections on the walls (see figure 1). Waveguides are implemented as soon as there is a strong attenuation upon propagation. For example, waveguides are used in radars, in air navigation and seamanship, telephony… The propagation of guided waves depend on various parameters, among which the geometry of the guide, the properties of the material constituting the guide as well as the wavelength of the guided waves.

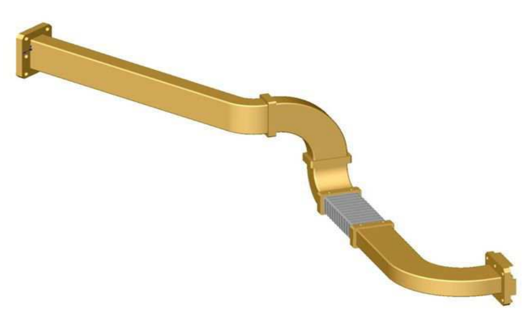


Figure 1: Example of a waveguide

Let's consider a waveguide made of metallic walls (perfect conducting medium), constituting a rectangular cross-section of inner dimensions a and b.
We choose to place the origin O of the coordinate system at a corner of the waveguide as indicated in Figure 2. We will consider that the medium located inside the waveguide is a perfect dielectric medium, uncharged (in other words ρ=0), of permeability μ_0 and permittivity ε.

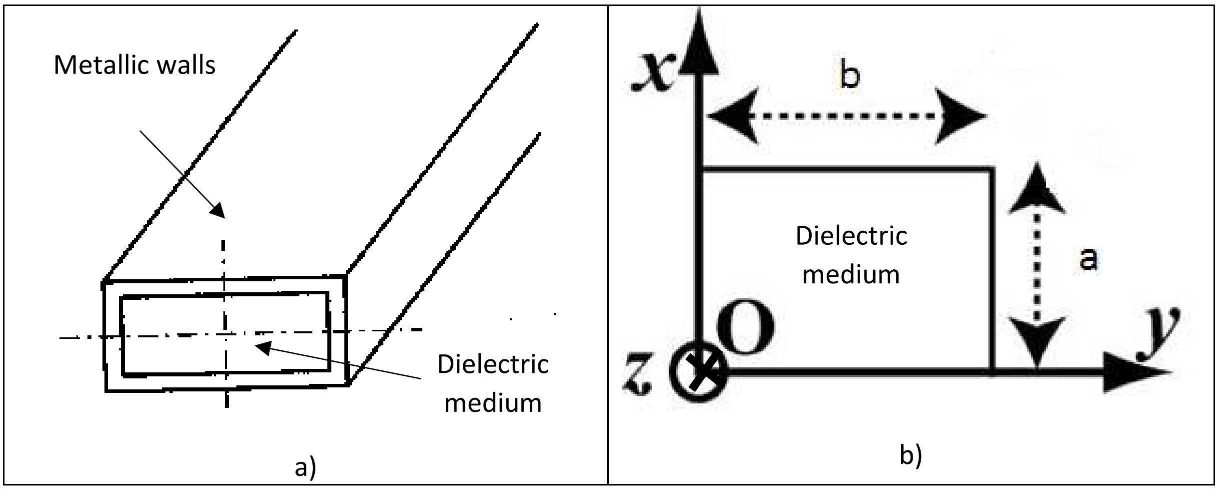


Figure 2 : a) geometry of a rectangular waveguide ; b) View of the section of the dielectric medium inside the waveguide, with the origin placed at a corner, note that the metallic walls surrounding the material are not represented.

We want an electromagnetic wave ($\vec{E},\vec{B}$) to propagate inside the waveguide, i.e. in the dielectric medium, whose electric field $\vec{E}$ is transverse harmonic (consequently its component along the direction z is nil) and of angular frequency ω. The electric field $\vec{E}$ has a random polarization in the xOy plane. The electromagnetic wave may be nonuniform. Moreover the waveguide is invariant with respect to any translation along Oz (its cross-section is everywhere the same). All these conditions lead to write the components of $\vec{E}$ and $\vec{B}$ in the form:

$$\underline{E_x} = \underline{f_x}(x,y).e^{j(\omega t - kz)} \qquad \underline{B_x} = \underline{g_x}(x,y).e^{j(\omega t - kz)}$$
$$\underline{E_y} = \underline{f_y}(x,y).e^{j(\omega t - kz)} \quad \text{and} \quad \underline{B_y} = \underline{g_y}(x,y).e^{j(\omega t - kz)}$$
$$\underline{E_z} = 0 \qquad \underline{B_z} = \underline{g_z}(x,y).e^{j(\omega t - kz)}$$

With $\underline{f_x}$, $\underline{f_y}$, $\underline{f_z}$, $\underline{g_x}$, $\underline{g_y}$, $\underline{g_z}$ functions that may be complex ones.

1. By using Maxwell's equations, deduce the equation of propagation (or wave equation) of the magnetic field $\vec{B}$ in the waveguide.
2. Starting with this equation, deduce 3 differential equations fulfilled respectively by $\underline{g_x}$, $\underline{g_y}$ and $\underline{g_z}$.
3. The resolution of the differential equation found for $\underline{g_z}$ indicates that this function can be written in the form $\underline{g_z} = B_0 \sin(\beta_1 x + \varphi_1)\sin(\beta_2 y + \varphi_2)$, with $\beta_1, \varphi_1, \beta_2, \varphi_2$ constants. φ_1 and φ_2 are comprised between 0 and π.

   From Maxwell-Ampere and Maxwell-Faraday equations, which enable to link $\vec{E}$ and $\vec{B}$, we deduce that:

$$jk_c^2 \underline{g_y} = k\frac{\partial \underline{g_z}}{\partial y} \qquad k\underline{f_y} = -\omega \underline{g_x}$$
$$\text{and} \qquad \qquad \text{with } k_c^2 = \omega^2 \mu_0 \varepsilon - k^2$$
$$jk_c^2 \underline{g_x} = k\frac{\partial \underline{g_z}}{\partial x} \qquad k\underline{f_x} = \omega \underline{g_y}$$

Deduce the expressions of $\underline{f_x}$, $\underline{f_y}$, as a function of $\beta_1, \varphi_1, \beta_2, \varphi_2$,B_0, k_c^2 and ω.

4. By applying <u>rigorously</u> the boundary conditions on the $\vec{E}$ field, find the values of φ_1 and φ_2 . Find also the expressions of β_1 and β_2 as a function of a, b and 2 integers m and n.
5. Deduce that the expression of the component $\underline{E_x}$ of the $\vec{E}$ field is :

$$\underline{E_x} = \frac{\omega B_0}{k_c^2} . \left(\frac{n\pi}{b}\right) . cos\left(\frac{m\pi x}{a}\right) . sin\left(\frac{n\pi y}{b}\right) . e^{j(\omega t - kz + \alpha)}$$

You'll give the value for α. The couple (m, n) defines what is called a mode.

6. Knowing that the electric field also verifies the equation $\Delta\vec{E} - \mu_0 \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$, show that
$$k^2 = \mu_0 \varepsilon \omega^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2$$
7. In the case where k^2 is negative, k is a pure imaginary number and can be written $k = -jk'$ (with k' a real number). Deduce the real expression of E_x. Characterize completely the wave associated with E_x.
8. We want that the wave propagates without attenuation. What is in this case the condition for k^2, why ?
9. What are the minimum values for m and n allowing a propagation of E_x. Deduce the existence of a cut-off angular frequency ω_c under which E_x will not propagate in the guide.
10. For a wave of angular frequency $\omega_{m,n} > \omega_c$ (corresponding to a mode (m,n)), give its phase velocity V_ϕ as a function of m, n , ω, a, b, μ_0, and ε. What can you conclude on the property of the media "dielectric in a guide". How could this be a problem?

Vectorial analysis formulae in Cartesian coordinates $(\vec{e_x}, \vec{e_y}, \vec{e_z})$:

$$\overrightarrow{\text{rot}}\left(\overrightarrow{\text{rot}}\,\vec{a}\right) = \overrightarrow{\text{grad}}\left(\text{div}\,\vec{a}\right) - \overrightarrow{\Delta a}$$

$$\overrightarrow{\Delta a} = \left(\frac{\partial^2 a_x}{\partial x^2} + \frac{\partial^2 a_x}{\partial y^2} + \frac{\partial^2 a_x}{\partial z^2}\right)\vec{e}_x + \left(\frac{\partial^2 a_y}{\partial x^2} + \frac{\partial^2 a_y}{\partial y^2} + \frac{\partial^2 a_y}{\partial z^2}\right)\vec{e}_y + \left(\frac{\partial^2 a_z}{\partial x^2} + \frac{\partial^2 a_z}{\partial y^2} + \frac{\partial^2 a_z}{\partial z^2}\right)\vec{e}_z$$

Maxwell equations in the general case:

$$\overrightarrow{\text{rot}}\vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \overrightarrow{\text{rot}}\left(\frac{\vec{B}}{\mu}\right) = \vec{j} + \varepsilon\frac{\partial \vec{E}}{\partial t}, \quad div\left(\varepsilon\vec{E}\right) = \rho, \quad div\vec{B} = 0$$

PROBLEM II : INTERFEROMETRIC RESOLUTION OF A BINARY STAR

In astronomy, a **binary star** is a stellar system consisting in two stars orbiting around a common center of mass. Sirius is a good example. The special interest of astronomers for binary stars is due to the fact that knowing their movements allows to calculate their mass and other stellar parameters (link to the presence of exo-planets). But the main difficulty for the astronomers is to distinguish the two stars when they are (seen from earth) very close to each other. One solution consists in using an interferometer device whose principle is described in this problem.

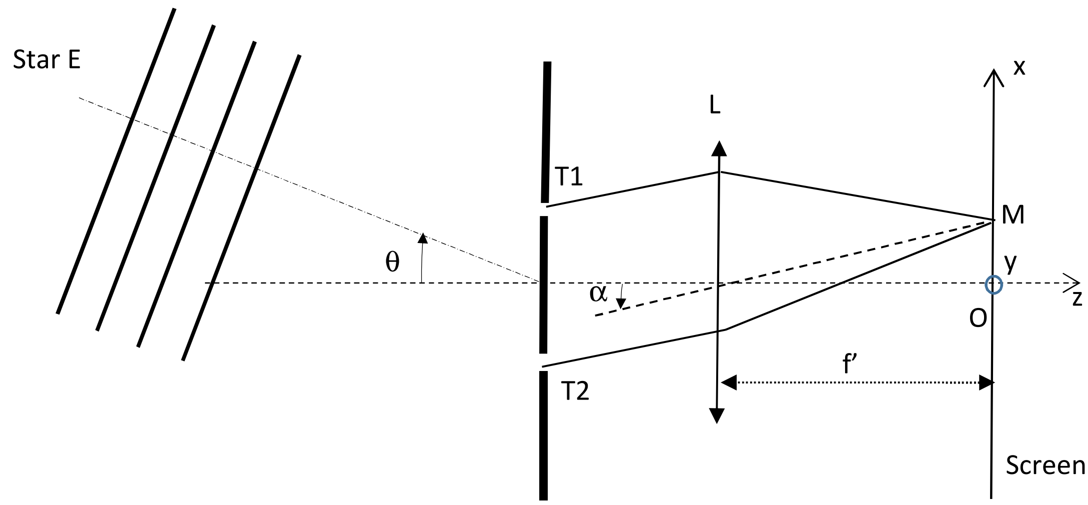


Figure 3 : Interferometric device lit by star E (plane wave with incidence angle θ).

The device is sketched on figure 3. It also includes a filter (not on the sketch) which lets through only radiations of wavelength in vacuum λ_v= 600 nm, and a plate with 2 holes in it T_1 and T_2 exactly identical, separated by a distance a, situated at equal distance from the optical axis of the setup, and so small they'll considered as points. A converging lens L of focal distance f' (f'=600 cm) and image focal point O receives the rays coming from T1 and T2. The observations are done on a screen situated at the focal distance of the lens. The position of a point M on a screen is located by its coordinates (x,y) in the Cartesian frame (O,Ox,Oy,Oz) defined on figure 3. We will make the (verified) assumption that f'>>a and f'>>x. All experiments will be conducted in air of refraction index n_{air}. By convention, the angle θ (between the optical axis and the incident ray) is counted positively for a clockwise orientation, and the angle α (between the optical axis and the out-coming ray) is counted positively for an anti-clockwise orientation: with this convention, **on figure 3, the angles α and θ are positive**.

PART A : device illuminated by a single star

In this first part, we will consider a single star E, located at a distance assumed infinite from the Earth. This star sends a plane progressive harmonic uniform wave, with an incidence angle θ from the (Oz) axis. θ is assumed very small (conversely to the sketch in figure 3).

1. Comment on the experimental device :

a) Describe the nature of the wave getting out of the holes T1 and T2 and justify. What can we observe in the focal plane of the lens L?

b) Why is it necessary to use a filter for λ_v (not shown in the figure) ?

2. Observation on the screen :

a) Prove that the optical path difference δ_2 at the point M(x,0) between two rays going through T_1 and T_2 and coming from E, can be written: $\delta_2 = n_{air} a \left(\theta + \frac{x}{f'} \right)$. From this expression, deduce the phase shift φ between these two rays.

b) What changes on the screen when θ increases?

c) Express $\underline{s_1}$ and $\underline{s_2}$ the complex amplitudes of the rays going through T_1, and T_2 respectively and arriving at point M (the maximal amplitude of the wave when it is on the screen will be denoted s_0). By a proof, deduce from this the intensity at point M along the axis Ox, as a function of a, x, f', λ_v, n_{air}, θ and I_0 (I_0 is the intensity that point M would have received if there had been only one hole).

d) Establish the literal expression of the interfringe, and calculate it for a=60.0 mm (with n_{air}=1)

e) What happens if the two holes T_1 and T_2 move further from each other?

Part B : device illuminated by two stars

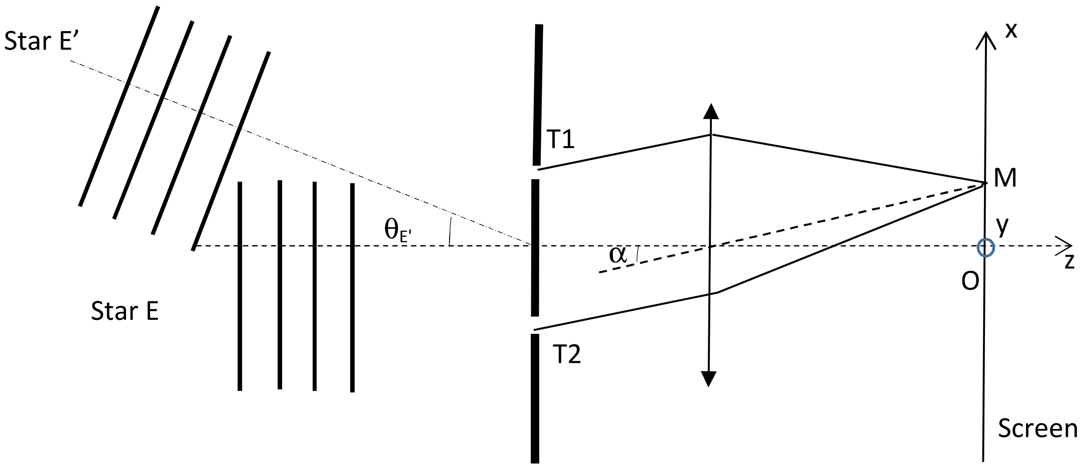


Figure 4: interferometric device illuminated by two stars E and E', separated by an angular distance $\theta_{E'}$ (the angles α et $\theta_{E'}$ are counted with the same convention as in figure 3)

The same device is now illuminated by two stars E and E', one with a zero incidence angle from the optical axis, the other with an incidence angle $\theta_{E'}$. It is said that both stars E and E', seen from the earth, are separated by an angular distance $\theta_{E'}$. $\theta_{E'}$ is very small, (conversely to its representation in figure 4).

3. The waves coming from E and E' are incoherent, why? What is the consequence on the light intensity received on the screen?
4. It is assumed that both waves emitted by both stars have the same intensity. Prove that the total intensity I_T received in the focal plane of the lens L, along the Ox axis, can be written: $I_T = 4I_0\left(1+\cos\left(\pi n_{air}\frac{a\theta_{E'}}{\lambda_v}\right)\cos\left(\pi n_{air}\frac{a}{\lambda_v}\left(\frac{2x}{f'}+\theta_{E'}\right)\right)\right)$.

Reminder : $\cos(p)+\cos(q) = 2\cos\left(\frac{p+q}{2}\right)\cos\left(\frac{p-q}{2}\right)$

5. What are the minimum value I_{min} and the maximal value I_{max} of I_T when x varies? By using them, deduce the expression of the contrast $\gamma = \frac{I_{max}-I_{min}}{I_{max}+I_{min}}$
6. What has to be the literal expression of *a* for the fringes to disappear? a_0 is its smallest value: give its expression.
7. Express the relative uncertainty on $\theta_{E'}$ as a function of the relative uncertainties on λ_v and a_0.
8. Numerical application: experimentally, a_0 is found equal to 1 m; with this value, calculate $\theta_{E'}$, the angular distance between both stars.

9. Actually, for a = a_0, the fringes do not disappear, even if their contrast is minimal, and is equal to γ_{min} = 0, 1. Which assumption among the ones made previously has to be reconsidered?

10. Bonus question: What is the interest of replacing both holes T_1 and T_2 by two slits F1 and F2 parallel to the Oy axis?

Part C : discussion about the device

We now compare the interest of the previous device to a telescope. The simplified scheme of the latter is depicted on figure 5: it has an aperture D (which is the diameter of the camera lens) and contains a lens placed at the focal distance f' from a screen.

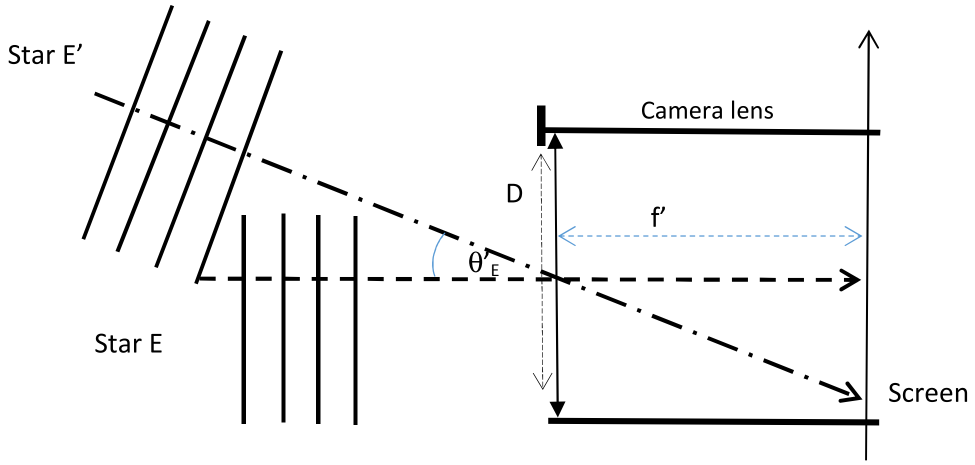


Figure 5: Scheme of a telescope illuminated by two stars E and E' separated by an angular distance $\theta_{E'}$.

11. What is the consequence of the presence of the aperture of diameter D on the rays coming from both stars?

12. The aperture strongly limits the angular resolution of the telescope, (in other words, it limits the angular distance between both stars, below which it is not possible to distinguish one star from the other anymore). Draw a scheme to explain why.

13. The Rayleigh criteria states that the images of two point sources are distinct if the distance between them is at least equal to the radius of the central diffraction spot; the latter one is delimited by the points where the diffracted intensity is nil (this minimum corresponds to $\sin\theta=1.22\lambda_v/D$, with θ the diffraction angle, and D the aperture diameter):

    a) What is the minimum angular distance between both stars E and E' so that they can be distinguished with a telescope which lens has a focal distance of 600 cm and which aperture diameter is 100 cm (characteristics of the "télescope du

Midi”). (Here, we still consider the monochromatic case with λ_v =600nm and n_{air}=1).

b) What would be the a_0 value (question 6 of part B) to get the same minimum angular distance using the previous device (figure 5)?

As proved in the problem, to separate double stars, the device in figure 4 is useful, compared to a usual telescope (whose resolution is limited by the Rayleigh criteria), only if the distance between both holes reaches several tens of meters. This can be practically impossible. Instead of placing two holes in front of the telescope lens, a usual method in astronomy then consists in using two (or even more) telescopes; this way, they can be separated by a distance which can reach several hundreds of meters. The interferences are created by the rays coming from the two telescopes, which play the role of the holes T1 and T2; and the distance between them is analogous to the distance a (for more details, see for instance www.f-legrand.fr).

Physique : IE longue Semestre 2

Mercredi 14 juin | Durée : 3h

CORRIGE

Exercice I : Etude d'un guide d'onde rectangulaire

1)	Justification que $\vec{j}=\vec{0}$ (et $\rho=0$) $\overrightarrow{rot}(\overrightarrow{rot}(\vec{B}))=\overrightarrow{rot}(\mu_0\varepsilon\frac{\partial\vec{E}}{\partial t})=\overrightarrow{rot}\mu_0\varepsilon\frac{\partial\overrightarrow{rot}\vec{E}}{\partial t}=-\mu_0\varepsilon\frac{\partial^2\vec{B}}{\partial t^2}$ $\overrightarrow{rot}(\overrightarrow{rot}(\vec{B}))=rot(div\vec{B})-\Delta\vec{B}=-\Delta\vec{B}$ d'où $\Delta\vec{B}-\mu_0\varepsilon\frac{\partial^2\vec{B}}{\partial t^2}=\vec{0}$
2)	$\Delta\vec{B}\begin{pmatrix}\left[\frac{\partial^2\underline{g}_x}{\partial x^2}+\frac{\partial^2\underline{g}_x}{\partial y^2}-k^2\underline{g}_x\right].e^{j(\omega t-kz)}\\\left[\frac{\partial^2\underline{g}_y}{\partial x^2}+\frac{\partial^2\underline{g}_y}{\partial y^2}-k^2\underline{g}_y\right].e^{j(\omega t-kz)}\\\left[\frac{\partial^2\underline{g}_z}{\partial x^2}+\frac{\partial^2\underline{g}_z}{\partial y^2}-k^2\underline{g}_z\right].e^{j(\omega t-kz)}\end{pmatrix}$ et $\frac{\partial^2\vec{B}}{\partial t^2}=-\omega^2.\vec{B}$ D'où $\begin{cases}\frac{\partial^2\underline{g}_x}{\partial x^2}+\frac{\partial^2\underline{g}_x}{\partial y^2}-k^2\underline{g}_x+\mu_o\varepsilon\omega^2\underline{g}_x=0\\\frac{\partial^2\underline{g}_y}{\partial x^2}+\frac{\partial^2\underline{g}_y}{\partial y^2}-k^2\underline{g}_y+\mu_o\varepsilon\omega^2\underline{g}_y=0\\\frac{\partial^2\underline{g}_z}{\partial x^2}+\frac{\partial^2\underline{g}_z}{\partial y^2}-k^2\underline{g}_z+\mu_o\varepsilon\omega^2\underline{g}_z=0\end{cases}$
3)	$\underline{f}_x=\frac{\omega}{k}\underline{g}_y=\frac{\omega}{k}\frac{k}{jk_c^2}\frac{\partial g_z}{\partial y}$ $\frac{\partial\underline{g}_z}{\partial y}=\beta_2B_0\sin(\beta_1x+\varphi_1)\cos(\beta_2y+\varphi_2)$ $\underline{g}_y=-j\frac{\beta_2B_0k}{k_c^2}\sin(\beta_1x+\varphi_1)\cos(\beta_2y+\varphi_2)$ $\underline{f}_x=-j\omega\frac{\beta_2B_0}{k_c^2}\sin(\beta_1x+\varphi_1)\cos(\beta_2y+\varphi_2)$

	De même $\underline{f}_y = +j\omega \frac{\beta_1 B_0}{k_c^2}\cos(\beta_1 x+\varphi_1)\sin(\beta_2 y+\varphi_2)$
4)	$\vec{E}=\vec{0}$ dans les parois métalliques car conducteur parfait (ou conductivité infinie) Continuité de la composante tangentielle du champ : $\begin{cases}\underline{f}_x = 0\ \forall t,\ \underline{\forall x} \text{ pour } \underline{y=0} \text{ et } \underline{y=b}\\ \underline{f}_y = 0\ \forall t,\ \underline{\forall y} \text{ pour } \underline{x=0} \text{ et } \underline{x=a}\end{cases}$ D'où $\varphi_1 = \varphi_2 = \frac{\pi}{2}$ Et $\beta_1 = \frac{m\pi}{a}; \beta_2 = \frac{n\pi}{b}$ avec m et n entiers
5)	$\underline{E}_x = -j\omega\left(\frac{n\pi}{b}\right)\frac{B_0}{k_c^2}\sin(\frac{m\pi x}{a}+\frac{\pi}{2})\cos(\frac{n\pi y}{b}+\frac{\pi}{2}).e^{j(\omega t-kz)}$ $= +j\omega\left(\frac{n\pi}{b}\right)\frac{B_0}{k_c^2}\cos(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b}).e^{j(\omega t-kz)}$ On retrouve l'expression de l'énoncé avec $\alpha = \frac{\pi}{2}$
6)	$\Delta E_x = \left[\frac{\partial^2 \underline{f}_x}{\partial x^2}+\frac{\partial^2 \underline{f}_x}{\partial y^2}-k^2 \underline{f}_x\right].e^{j(\omega t-kz)} = \left[-\left(\frac{m\pi}{a}\right)^2-\left(\frac{n\pi}{b}\right)^2-k^2\right].\underline{f}_x.e^{j(\omega t-kz)}$ et $\frac{\partial^2 \underline{E}_x}{\partial t^2} = -\omega^2.\underline{E}_x$ d'où $-\left(\frac{m\pi}{a}\right)^2-\left(\frac{n\pi}{b}\right)^2-k^2+\mu_0\varepsilon\omega^2 = 0$
7)	$\underline{E}_x = \frac{\omega B_0}{k_c^2}\left(\frac{n\pi}{b}\right)\cos(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b}).e^{-k'z}.e^{j(\omega t+\frac{\pi}{2})}$ D'où l'expression réelle : $E_x = \frac{\omega B_0}{k_c^2}\left(\frac{n\pi}{b}\right)\cos(\frac{m\pi x}{a})\sin(\frac{n\pi y}{b}).e^{-k'z}.\cos(\omega t+\frac{\pi}{2})$ Onde <u>amortie</u> (accepter avec atténuation), <u>pas de propagation</u> (accepter stationnaire, ou onde évanescente)
8)	Propagation sans atténuation : il faut garder le terme $e^{j(\omega t-kz)}$ Il faut donc k réel, c'est-à-dire $k^2 > 0$

9)

$$k^2 > 0 \Leftrightarrow \omega > \sqrt{\frac{1}{\mu_0 \varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

La valeur minimale de $\sqrt{\frac{1}{\mu_0 \varepsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$ est obtenue pour (m,n)=(0,0)

Mais n=0 n'est pas acceptable car sinon on aurait $E_x = 0$

La valeur minimale acceptable est donc (m,n)=(0,1)

Alors $\omega_c = \frac{\pi}{b\sqrt{\mu_0 \varepsilon}}$

10)

Pour une onde de pulsation $\omega_{m,n} > \omega_c$, $k_{m,n} = \sqrt{\mu_0 \varepsilon \omega^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}$

D'où $V_\varphi = \frac{\omega}{k_{m,n}} = \frac{\omega}{\sqrt{\mu_0 \varepsilon \omega^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2}}$

Le milieu est dispersif. L'information qui peut contenir plusieurs fréquence transportée dans le guide d'onde risque d'être déformée (ou toute formulation équivalente)

EXERCICE 2 : Etoile double

A.1.a)

Nature des ondes : ondes sphériques car diffraction par un trou ponctuel

On observe l'interférence des deux ondes sorties de T1 et de T2, donc

un système de franges rectiligne, parallèle à O_y

(c'est une approximations puisqu'en réalité, on a le résultat de l'intersection d'un hyperboloide avec le plan)

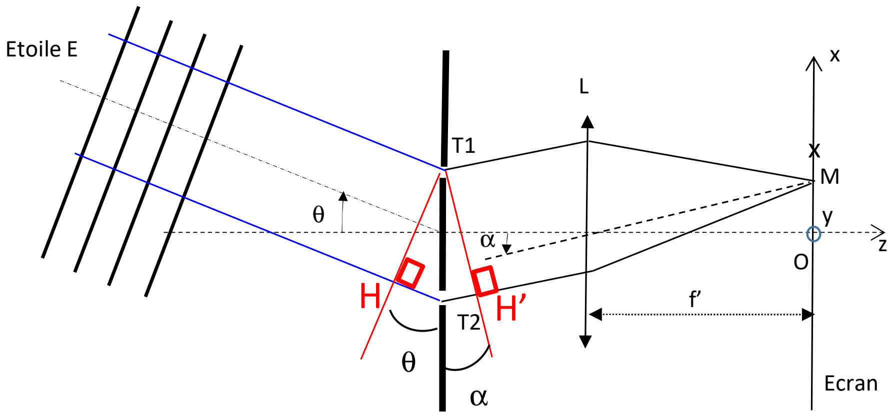
A.1.b)

Le filtre en l permet de travailler en lumière monochromatique

Sinon, il va y avoir superposition des différents systèmes de franges dus aux différentes longueur d'ondes, ces derniers ont des distance interfranges différentes, il y aura donc « brouillage » de la figure d'interférence.

A.2.a)

Figure : Source E à l'infini faisant un angle θ très petit avec l'axe optique



$$\delta = n_{air}((ET_2 + T_2M) - (ET_1 + T_1M)) = n_{air}(HT_2 + T_2H')$$

$\hat{\theta} = T_2\hat{T}_1H$ et $\hat{\alpha} = T_2\hat{T}_1H'$ donc $HT_2 = T_1T_2 \sin\theta \approx a\theta$ et $T_2H' = T_1T_2 \sin\alpha \approx a\alpha$

$\delta = a(\theta + \alpha) = a(\theta + x / f')$ avec $\tan\alpha = x / f' \approx \alpha$

Le déphasage est : $\varphi = 2\pi \dfrac{\delta}{\lambda_v} = \dfrac{2\pi}{\lambda_v} n_{air} a(\theta + \dfrac{x}{f'})$

A.2.b)

Si θ augmente, δ augmente (ou φ augmente).
Pour avoir la même valeur de delta, correspondant à une frange donnée, il faut compenser l'augmentation de θ par une diminution de x.
Donc les franges se décalent vers le bas

A.2.c)

$\underline{s_1}(M,t) = s_0 e^{j\varphi_0} e^{j(\omega t - k r_1)}$ et $\underline{s_2}(M,t) = s_0 e^{j\varphi_0} e^{j(\omega t - k r_2)}$

$\underline{s_T} = \underline{s_1} + \underline{s_2}$

avec r1 et r2 les chemins géométriques des deux rayons issus de E et φ_0 la phase aléatoire à l'origine.

$$I = \underline{s_T}(P,t) \times \overline{\underline{s_T}(P,t)} = s_0^{\,2} e^{j\omega t} e^{-j\omega t}\left[e^{-jkr_1} + e^{j(-kr_2)}\right] \times \left[e^{+jkr_1} + e^{-j(-kr_2)}\right]$$

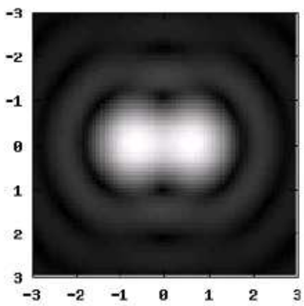
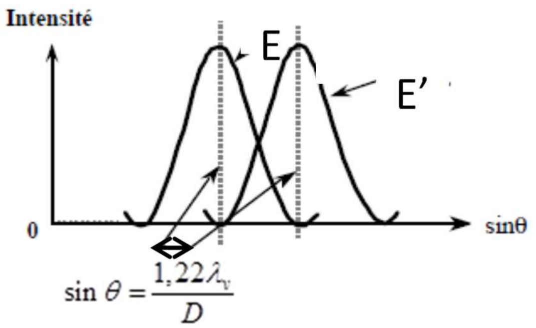
$$I = s_0^{\,2}\left[1 + e^{+j(kr_2 - kr_1 - \pi)} + e^{-j(kr_2 - kr_1 - \pi)} + 1\right] = 2I_0\left(1 + \cos\left(k\left(r_2 - r_1\right)\right)\right)$$

$$= 2I_0\left(1 + \cos\left(k\delta\right)\right)$$

(Ou bien $I = 4I_0\left(\cos^2\left(\dfrac{k\left(r_2 - r_1\right)}{2}\right)\right) = I_0\left(\cos^2\left(\dfrac{k\delta}{2}\right)\right)$

$$I = \underline{s_{tot}}\,\underline{s_{tot}}^* = 2I_0\left(1 + \cos\left[\frac{2\pi}{\lambda_v} n_{air} a(\theta + \frac{x}{f'})\right]\right)$$

A.2.d)	I max pour $\frac{2\pi}{\lambda_v} n_{air} a(\theta+\frac{x}{f'}) = 2p\pi$ L'interfrange se calcule via la difference de phase entre la $p^{ième}$ et la $p+1^{\,ième}$ frange maximale qui vaut 2π : $\frac{2\pi}{\lambda_v} n_{air} a(\theta+\frac{x+(p+1)i}{f'}) - \frac{2\pi}{\lambda_v} n_{air} a(\theta+\frac{x+pi}{f'}) = 2\pi$ soit : $i = f'\frac{\lambda_v}{n_{air}a}$ AN : i =60µm
A.2.e)	Si a augmente, i diminue , les franges se rapprochent – il faut un occulaire pour grossir l'observation
B.3)	E et E' sont des sources (étoiles) indépendante différentes, elles sont donc incohérentes ⇨ Pas d'interférences et donc intensité sur l'écran égal à la somme des intensités dues à chacune des étoiles.
B.4)	$I_T = I + I' = 2I_0\left(2+\cos\left[\frac{2\pi}{\lambda_v} n_{air} a \frac{x}{f'}\right] + \cos\left[\frac{2\pi}{\lambda_v} n_{air} a(\theta_{E'} + \frac{x}{f'})\right]\right)$ $I_T = I + I' = 4I_0(1+\cos\left[\pi n_{air}\frac{a\theta}{\lambda_v}\right]\cos\left[\pi n_{air}\frac{a}{\lambda_v}(\theta+\frac{2x}{f'})\right])$
B.5)	I_T est une fonction de x dont les extrema sont obtenus lorsque le second cosinus vaut ±1 $\mathrm{I}_{max} = 4I_0\left(1+\left\lvert\cos\left[\pi n_{air}\frac{a\theta}{\lambda_v}\right]\right\rvert\right)$ $\mathrm{I}_{min} = 4I_0\left(1-\left\lvert\cos\left[\pi n_{air}\frac{a\theta}{\lambda_v}\right]\right\rvert\right)$ $\gamma = \left\lvert\cos\left[\pi n_{air}\frac{a\theta}{\lambda_v}\right]\right\rvert$
B.6)	γ s'annule pour c'est-à-dire pour $\pi\frac{n_{air}a\theta}{\lambda_v} = (2p+1)\frac{\pi}{2}$ soit $a_p = (2p+1)\frac{\lambda_v}{2\theta n_{air}}$ $a_0 = \frac{\lambda_v}{2\theta n_{air}}$
B.7)	. $\frac{\Delta\theta_{E'}}{\theta_{E'}} = \frac{\Delta\lambda_v}{\lambda_v} + \frac{\Delta a_0}{a_0}$
B.8)	$\theta_{E'} = \frac{\lambda_v}{2n_{air}a_0}$ => $\theta_{E'} = \frac{6.10^{-7}}{2} = 3.10^{-7}$ rad
B.9)	L'hypothèse de sources (étoiles) de même intensité
B.10)	On a alors ici l'équivalent de la diffraction à deux fentes pour chacune des étoiles. Cela doit donc donner des taches plutôt que des franges. L'intensité étant concentrée dans ces tâches, elle est donc augmentée. Mettre 1 point pour tout commentaire pertinent, MAIS ne pas mettre de point pour une réponse parlant d'incohérence car la fente n'est pas la somme de points sources incohérents puisqu'elle est éclairée par une onde plane.

C. 11)	La présence d'une ouverture conduit au <u>phénomène de diffraction.</u>
C.12)	Schéma : par exemple deux figures de diffraction s'interpénétrant ou toutes explications intelligibles équivalentes avec schéma  
C.13. a)	$\sin\theta = \frac{1,22\lambda_v}{D}$ donc $\sin\theta = \frac{1,22.600.10^{-9}}{1} = 7,32.10^{-7} \approx 7,3.10^{-7}\, rad \Rightarrow \theta \approx 7,3.10^{-7}\, rad$
C.13. b)	$a_0 = \frac{\lambda_v}{2\theta}$ => $a_0 = \frac{600.10^{-9}}{2.7,3.10^{-7}} = 0,41m$