INSA

2nd Year Physics written exam n° 2 Thursday 1 December 2016

Duration 1h30. Permitted Document: two-sided, personal, hand-written Synopsis Sheet. Any form of Calculator.

The three Parts are largely independent of each other. However, notions and concepts useful in Part 3 may appear in Part 2. Approximate marking scheme: Part 1, 4 pts; Part 2, 7 pts; Part 3, 9 pts.

Part I: Calculation of the current in a washing machine motor

We will model the motor of a washing machine by a series (R, L) circuit with an applied voltage $v(t) = \sqrt{2}V_{eff}\cos(\omega t)$ supplied by the French domestic network, EDF.

1.1- Write, without solving for the moment, the differential equation, valid once the circuit is closed (at t=0), which the current *i(t)* flowing through the circuit verifies.

1.2- What is the value of the current $i(0^+)$ in the circuit once the voltage is applied ? Justify clearly your answer.

1.3- Given that under steady state conditions one can express the current in complex notation as $\underline{i(t)} = I_0 e^{j(\omega t+\psi)}$, determine I_0 and ψ and express the real current i(t) under steady state conditions.

1.4- Give the expression of *i(t)*, the solution of the differential equation established in 1.1, from the closure of the circuit (t=0) - bearing in mind the initial condition stated in question 1.2 and that the circuit is subjected to a voltage $v(t)$.

1.5- What are the numerical values of V_{eff} and the angular frequency? (*bonus question*)

Part II :Risk of electric shock resulting from insulation defect

Household electric supply involves three wires: a live wire (of red or brown color),a wire for the neutral (of blue color) and a wire for the earth (of green and yellow color). Various household appliances, such as the motor of a washing machine, are connected in parallel to the live and neutral wires. The casing of the washing machine is connected to the earth wire. When one's hand is in contact with the casing of the washing machine and one's feet are in contact with the floor, we will represent that electrical circuit by a simple "human" resistance R_h whose value is found to depend on the voltage applied across it (figure 1.e). The resistance of the ground wire is modelled by a resistance R_a.

In normal use, we will assume that the situation described in figure 1.a can be modelled by the electrical circuit given in figure 1.b, where the switch K is open (the impedance Z represents the motor of the washing machine).

The two following questions do not require any calculation.

2.1 What can we say about the currents $i_P(t)$ and $i_N(t)$ circulating respectively in the live and neutral wires?

2.2 What can we say about the current $i_T(t)$ circulating in the earth wire ?

A fault in the washing machine, namely a defective insulation, can lead to a high voltage appearing on the casing.

This phenomenon can occur, for example, if the insulating sheath around the live wire becomes deteriorated and the uninsulated live wire then comes in contact with the metallic casing of the machine. Someone, in contact with the floor and touching the casing with their hand will then be subject to a significant voltage (figure 1.c).

The electrical circuit of figure 1.d describes this situation. Calculations will involve the use of complex notation. We will denote $\underline{i_h}$ the current flowing to earth through the human body and $\underline{i_c}$ that flowing to earth through the casing of the washing machine.

2.3 a) Give the relationship which exists between the voltages $\underline{u_{BE}}$ and $\underline{u_{CD}}$.

b) Deduce a relationship linking the currents $\underline{i_h}$ and $\underline{i_c}$.

c) What can we say about the above relationship if $R_h \gg R_a$? This approximation can be used later in the exercise if needed.

2.4 a) What is the relationship existing between $\underline{i_T}$, $\underline{i_c}$ and $\underline{i_h}$?

b) Deduce a simplified expression of $\underline{i_T}$ as a function of $\underline{i_c}$.

2.5 a) By using the current loop ABEFGA, express $\underline{i_c}$ as a function of $\underline{v}$.

b) Deduce $\underline{i_h}$ as a function of $\underline{v}$.

2.6 Complete the following table by calculating numerically the r.m.s. values of the intensities $\underline{i_c}$ and $\underline{i_h}$ for various r.m.s voltages V_{eff} . You will take R_a=10 Ω and make use of figure 1.e.

V_{eff} (V)	10	50	230
$I_{h\ eff}$ (mA)			
$I_{c\ eff}$ (A)			

2.7 With the help of Table 1 of document 2, explain when and in which way this situation can be dangerous for human beings.

The phenomenon observed above is used in the design of a device to protect people against insulation defects; it is called a residual-current circuit breaker.

Part III : Study of the functioning of a residual-current circuit breaker

We will use the cylindrical coordinate system ($\vec{e_r}$, $\vec{e_\theta}$, $\vec{e_z}$). Let's consider a torus of axis zz', of circular section, built of a perfect (i.e. LHI) magnetic material of permeability μ which channels the field lines. We denote by r_i the interior radius of the torus and r_e it's exterior radius. Over the whole torus, we wind a conducting wire (sheathed by an insulator), so that it forms N adjoining loops. A sinusoidal current i(t) is flowing through this wire, but we will assume that magnetostatic laws are still valid. We will only consider here the magnetic field $\vec{B}$ inside the torus (we will assume that the magnetic field is nil outside the torus).

3.1 a) Make a sketch showing the convention chosen for the orientation of the intensity.

b) Study and detail rigorously the topography of $\vec{B}$ inside the torus.

c) Establish with a detailed justification the expression of $\vec{B}$ using integral form of Ampere's Law.

d) We will consider that if r_i and r_e are close enough, the norm of the $\vec{B}$ field inside the torus can be considered uniform and has approximately the value calculated using the expression of question 3.1.c with $r = \frac{r_e+r_i}{2}$. Deduce this approximate expression.

*Figure 2 gives the scheme of a residual-current circuit breaker. Part of the live wire and of the neutral wire are wound on the torus, making 2 coils of **N_d** loops each, and creating a magnetic field respectively $\overrightarrow{B_P}$ and $\overrightarrow{B_N}$ in the torus when a current **i_P**(t) and **i_N(t)** is flowing through them. We will assume that the **direction** (but not necessarily the orientation) of the magnetic field lines of $\overrightarrow{B_P}$ and $\overrightarrow{B_N}$ are the same as for field $\vec{B}$ and that their **norms, denoted, B_P and B_N**, can be calculated using the expression established in 3.1.d., modifying the name of some of the variables.*

3.2 Taking into account the orientation of the currents i_N and i_P, derive the expression of the total field $\overrightarrow{B_t}$ created by these currents at a point M in the torus as a function of B_P, B_N and $\overrightarrow{e_\theta}$ and put it in the form:
$\overrightarrow{B_t} = A(i_N - i_P)\overrightarrow{e_\theta}$, specifying the expression of A.

We wind a third wire making N_m loops around the torus, and we measure with a voltmeter the voltage V(t) between the two ends of this wire (see Figure 2).

3.3 a) There is a defective insulation. Justify that there is a voltage V(t) across the voltmeter.

b) Determine the expression of V(t) as a function of the courant i_T(t) (defined in part 2).

c) What would the voltmeter indicate in normal use (no insulation defect) of the washing machine?

3.4. Using document 2, in how much time should the device linked to the voltmeter cut off the electrical power supply in order to guarantee an optimal protection to the user if a current of 30mA is flowing through him?

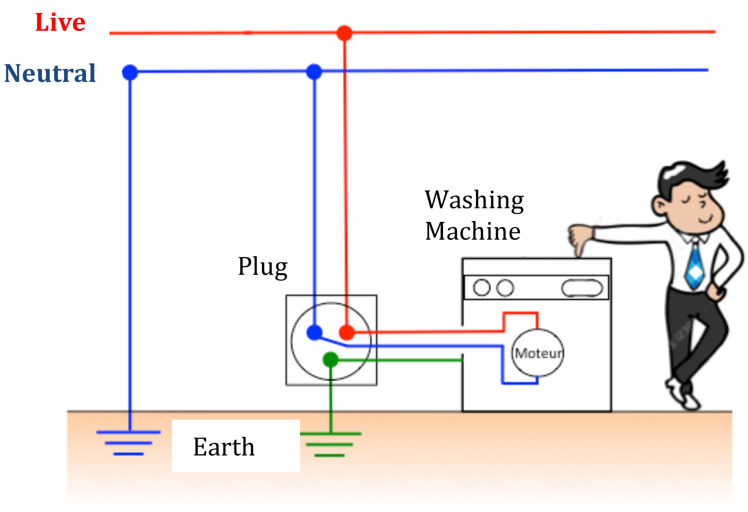

Figure 1a: Person touching casing of washing machine during normal functioning.

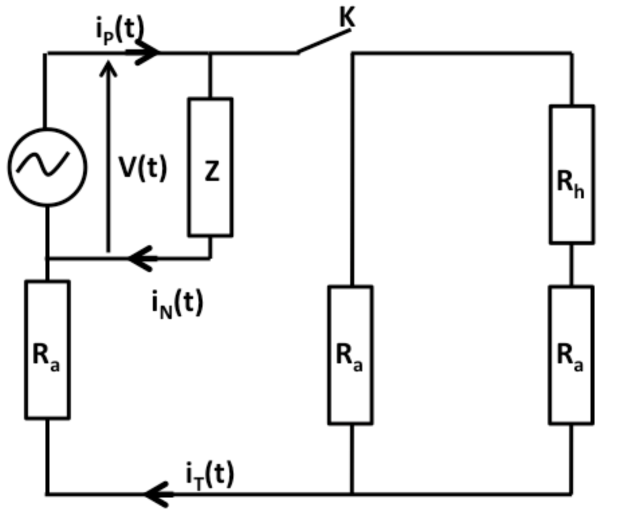

Figure 1b: Equivalent circuit diagram of the situation in figure 1.a

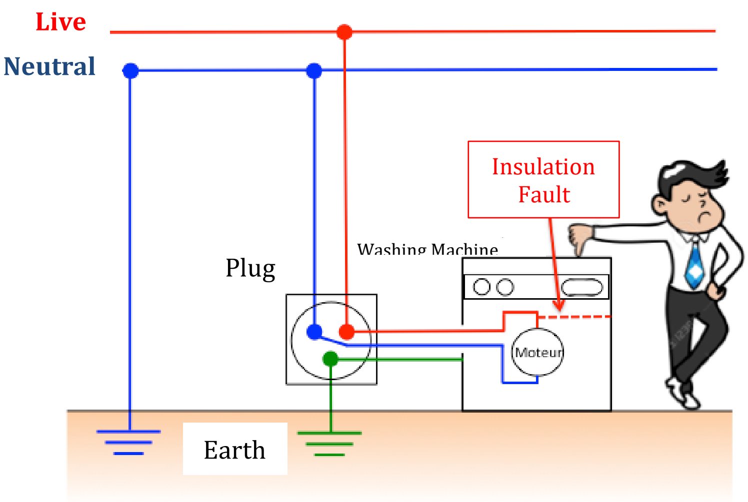

Figure 1c: Person touching casing of washing machine in presence of an insulation defect.

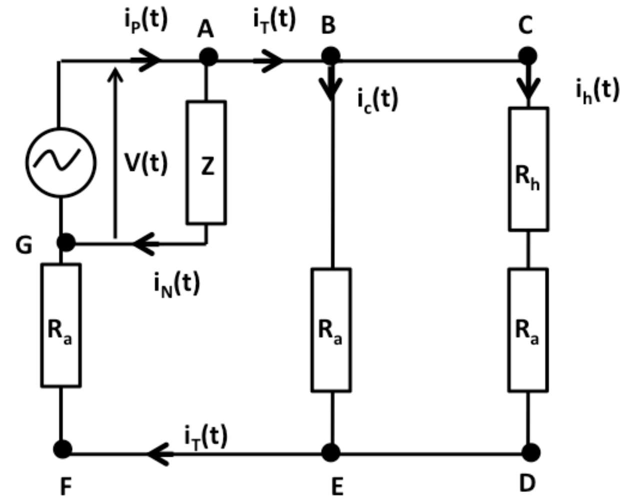

Figure 1d: Equivalent circuit diagram of the situation in figure 1.c with insulation defect.

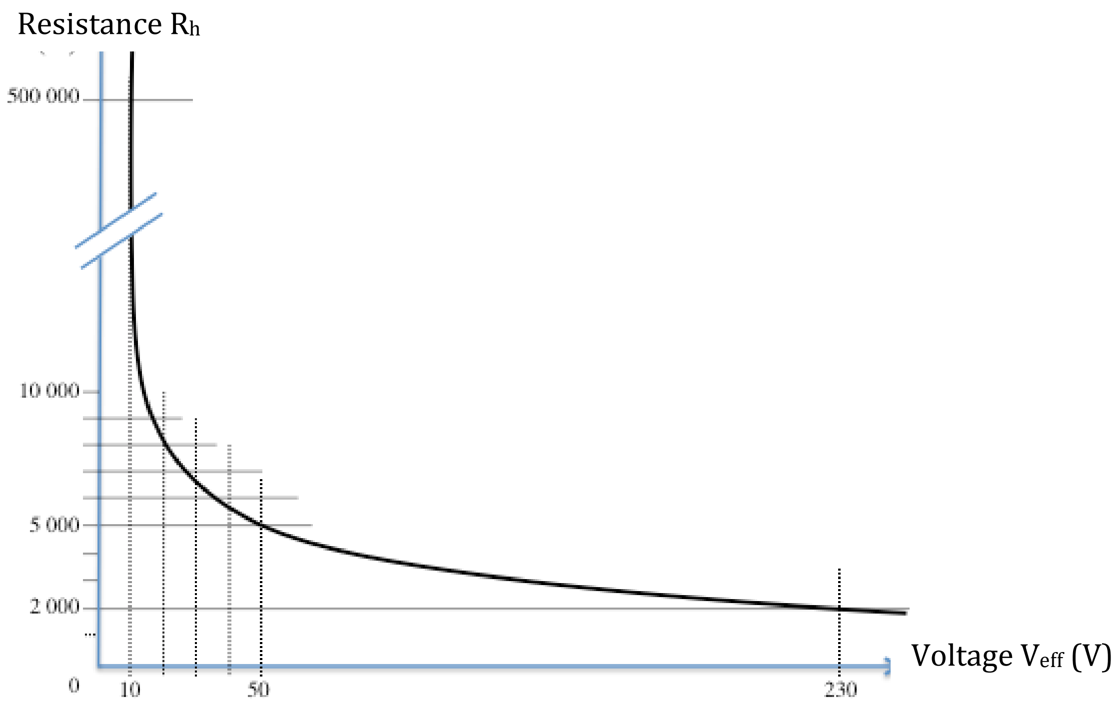

Figure 1.e: Resistance, R_h, of human body as a function of voltage.

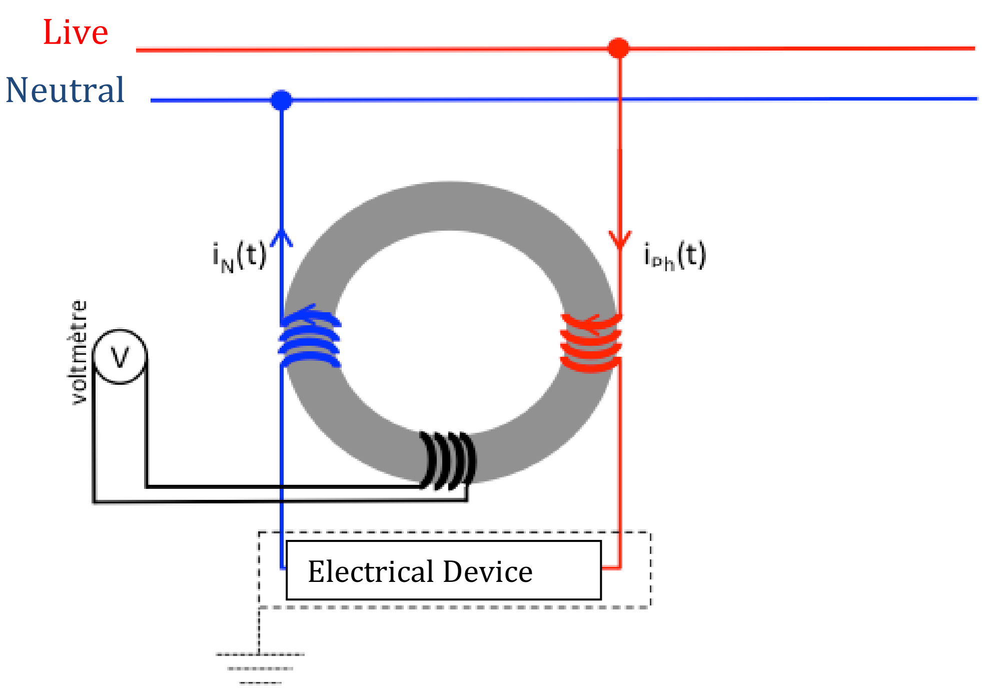

Figure 2: Diagram illustrating the principle of a residual-current circuit breaker

DOCUMENT 2: EFFECTS OF ELECTRICAL CURRENT ON THE HUMAN BODY

Current Amplitude	Effects on human body
0.5 mA	No sensation
1 mA	Threshold of perception
3 à 10 mA	Painful Sensation
10 mA	Muscular tetanisation: Threshold
30 mA	Respiratory paralysis, threshold
50 mA	Oxidation of blood
75 mA	Threshold of Cardiac Fibrillation
500 mA	Cardiac Fibrillation with probability >50 %
1 A	Burning of Nervous System
3 A	Cardiac Arrest
5 A	Irreversible internal burns

Table 1: Effect of current on the human body as a function of amplitude.

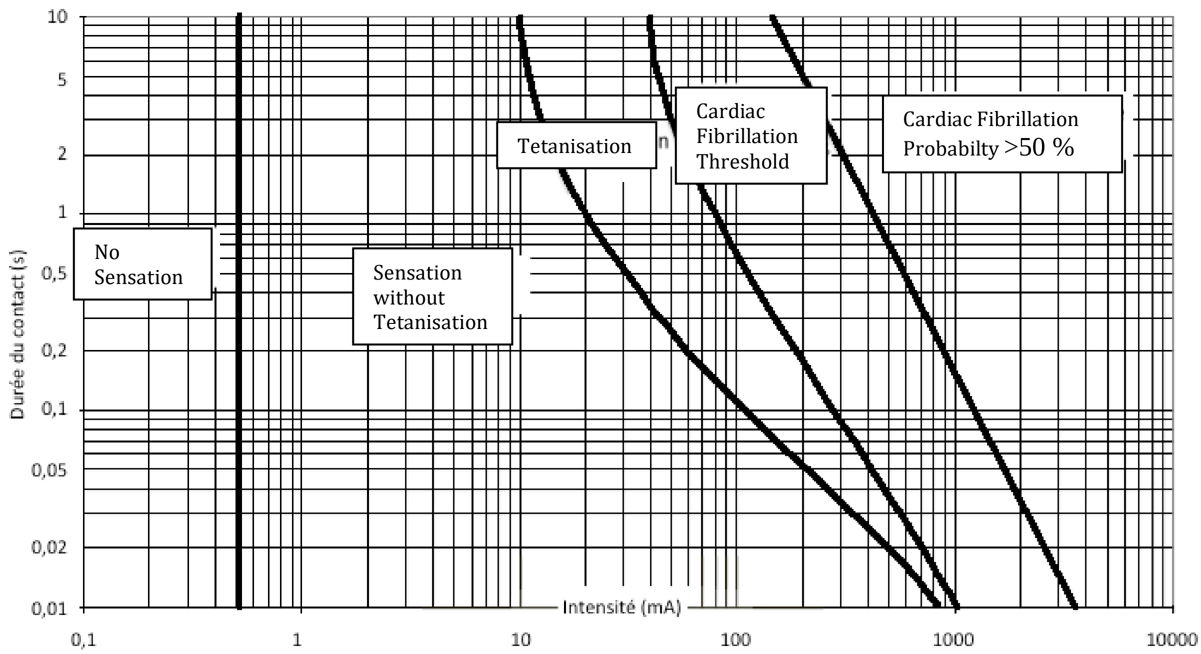

Figure 3:Effects of the electrical currents as a function of their intensity and the time of contact..