The subject comprises four independent exercises. The given marking scheme is only tentative.
Documents permitted: one handwritten sheet, written on one side
High-school type calculator authorized

Formulae: vector operators in cylindrical coordinates:

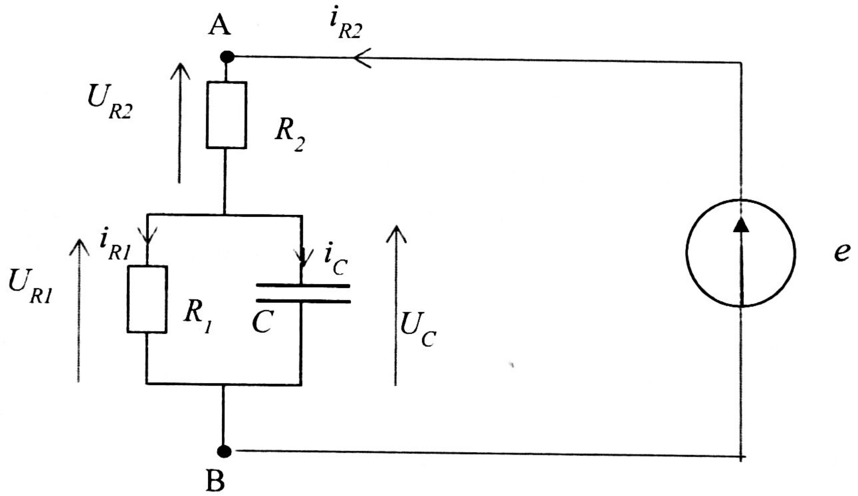
For $\vec{A} = A_r\overrightarrow{u_r} + A_\theta\overrightarrow{u_\theta} + A_z\overrightarrow{u_z}$

$$div(\vec{A}) = \frac{1}{r}\left(\frac{\partial(rA_r)}{\partial r} + \frac{\partial A_\theta}{\partial \theta} + \frac{\partial(rA_z)}{\partial z}\right)$$

$$rot(\vec{A}) = \frac{1}{r}\left(\frac{\partial A_z}{\partial \theta} - \frac{\partial(rA_\theta)}{\partial z}\right)\vec{u_r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{u_\theta} + \frac{1}{r}\left(\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta}\right)\vec{u_z}$$

Exercise 1: Electrocinetics (6 points)

In the electric circuit sketched below, the e.m.f. of the ideal voltage supply is $e = e_o\cos(\omega t+\phi)$



1/ The switch is closed at t=0, the capacitor being initially discharged. Give and justify the values of U_C, U_{R1}, U_{R2}, i_C, i_{R1}, i_{R2} at t=0+.

2/ In the forced sinusoidal regime, determine the complex impedance of the circuit between A and B.

3/ Establish the expression of the transfer function $\underline{H}(j\omega) = \frac{\underline{U_{R2}}}{\underline{e}}$. Calculate its modulus and its argument.

4/ We will assume in this question that $R_1 = 100\ R_2$. What is the expression of $\underline{H}(j\omega)$ when ω tends to zero and when ω tends to infinity? Deduce which type of filter this is. You may assume that the modulus is a monotonous function of ω.

Exercise 2: Differential operators and Maxwell equations (5 points)

We study this exercise using a system of cylindrical coordinates.

Consider the radial field: $\vec{D} = D_0\left(1 - \frac{z}{a}\right)\vec{u_r}$ where D_0 and a are strictly positive constants.

1/ Calculate its rotational and its divergence.

2/ Could this field be an electric field? Could it be a magnetic field? Justify your answers.

Consider now the field: $\vec{F} = F_0\left(1-\frac{z}{a}\right)\vec{u}_z - F_0\frac{\overset{r}{\cancel{x}}}{2a}\vec{u}_r$ where F_0 and a are strictly positive constants.

3/ Calculate its rotational and its divergence.

Exercise 3: Topography of a magnetic field (3 points)

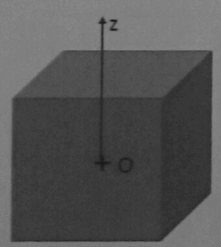
A solenoid of finite length is formed from an electric wire wound around an insulating cylinder, of axis Oz, radius R and delimited by planes z = -L/2 and z = + L/2. A current of amplitude I is flowing through the coiled wire.

1) Sketch the arrangement
2) Determine the topography of the resultant magnetic field, without performing any calculations but in justifying clearly your answer, in terms of direction(s) and spatial variables upon which it may depend
    a) at a point in space M(r,θ,z)
    b) at a point P(r,θ,0)
    c) at a point Q(0,0,z)

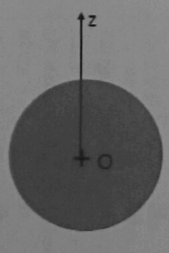
Exercise 4: Calculation of electric field (6 points)

Given a distribution of charges extending to infinity between the planes z = -a and z = +a with a volume charge density ρ(z), and none anywhere else in space

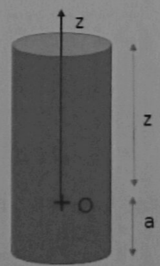
1) Determine the topography of the electric field $\vec{E}$ created throughout all space.
2) Given that $\rho(z) = \rho_1 \cos(\pi\frac{z}{2a})$:
    a) what can be said concerning $\vec{E}$ at two points M and M' which are symmetrical about the plane z = 0 ?
    b) what is the magnitude of the $\vec{E}$ field in the plane z = 0 ?
    c) for each of the following surfaces, indicate if they are suitable to be used to calculate $\vec{E}$ at point M(x,y,z) using Gauss' law. Cleary justify your answers *(no need to calculate E here)*.



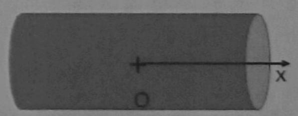
A: Cube of centre O
of sides 2z



B: Sphere of centre O
Of radius z

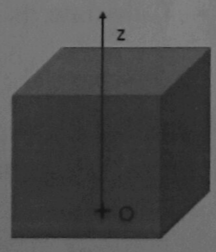


C: Cylinder of axis (Oz), not centred
on O, of height z+a

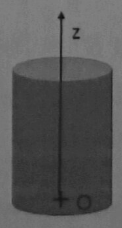


D: Cylinder of axis ~~(Oz)~~, (Ox)
centred on O, of length 2z

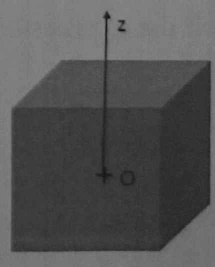
E: Straight prism of axis (Oz), with O on one
corner of base, height z



F: Cube with O on the base,
of sides z



G: Cylinder of axis (Oz), O on base of
cylinder of height z



H: Cube of centre O,
of sides 2a

3) Calculate $\vec{E}$ in the different regions of space using local relations. Clearly justify each step of your calculation.