Physics : Examination n°3

Monday March 5th Duration : 1h30

Indicative marking scheme : ***Lecture questions on 2 points, exercise 1 on 10 points, exercise 2 on 8 points.***

The exam is made of two independent exercises

Authorized document: one synthesis sheet RECTO, original and handwritten

Authorized calculator: any type but not internet connected, mobile phone not allowed

Lecture questions :

1. What are the necessary conditions for electromagnetic waves to create interferences?
2. We consider a rigid acoustic tube having at x=0 one end closed and at x=L one open end (see Figure 1). For the two first vibration modes plot schematically the evolution of the overpressure as a function of x in the tube. No justification is expected.

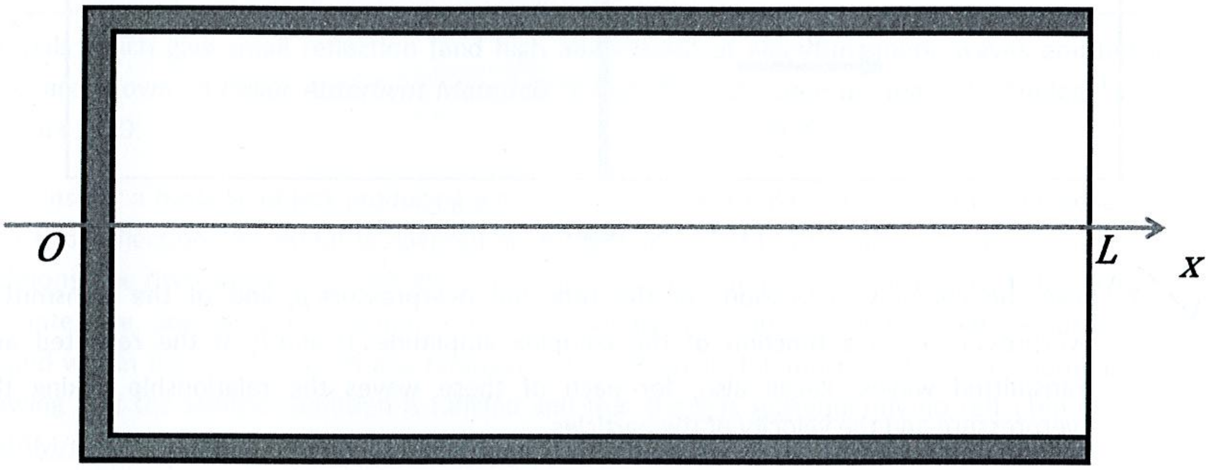

Figure 1

Exercise 1 :

In the absence of overpressure, the air of your apartment and of your neighbor's is at the ambient pressure P_o. A plane, progressive, harmonic acoustic wave of angular frequency ω propagates in air along increasing x in your neighbor's apartment (see Figure 2). By using the complex notation, the overpressure of amplitude P_i, can be expressed as:

$$\underline{p_i}(x,t) = P_i . e^{j(\omega t - kx)}$$

This wave reaches under normal incidence the wall separating your neighbor's apartment from yours. We will admit that this wall can be modelled by a piston of mass M, of surface S, **not deformable and of negligible thickness** (see Figure 2). In reality the wall behaves like an elastic membrane that comes back to its equilibrium position in case of lack of overpressure. To take this feature into account in the modelling, we will consider that the piston is submitted to a restoring force of the type:

$$\vec{F}(t) = -Ku_m(t)\vec{e_x}$$

where K is a positive constant. The restoring force tends to bring the wall back to its rest position (at $x = x_0 = 0$) when a displacement $u_m = x - x_0$ along $\overrightarrow{e_x}$ is applied to it. Under the action of the wave we will consider that the piston oscillates around its rest position, following a displacement $\underline{u_m}$ of the form:

$$\underline{u_m} = U_0 . e^{j\omega t}$$

In this exercise we want to characterize the wave that is transmitted in your apartment. We will denote c the velocity of sound in air, Z the air impedance, and k the wave number of the sound wave in air.

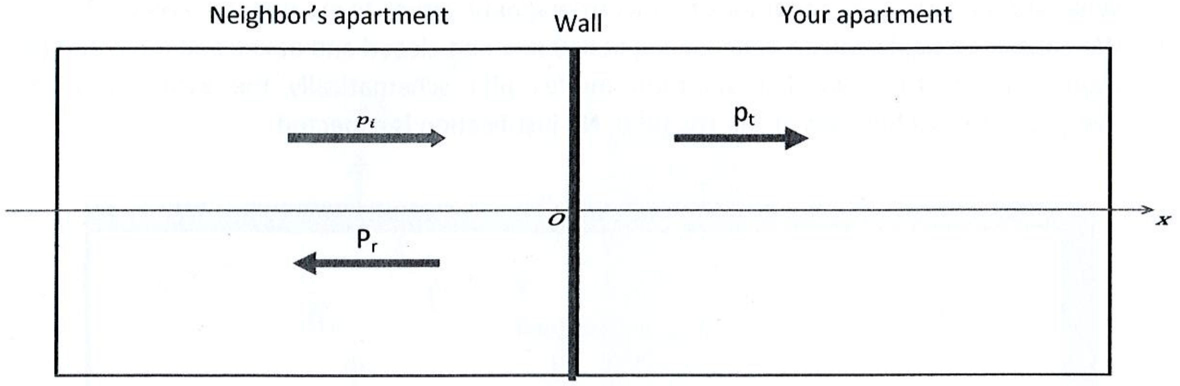

Figure 2

1. Give the complex expressions of the reflected overpressure $\underline{p_r}$ and of the transmitted overpressure $\underline{p_t}$ as a function of the complex amplitudes $\underline{P_r}$ and $\underline{P_t}$ of the reflected and transmitted waves. Recall also, for each of these waves the relationship linking the overpressure and the velocity of the particles.

2. Boundary conditions :
   a. With the help of a sketch consider all the forces exerted on the wall. **Do pay a particular attention to the orientation of the forces applied to the wall.** Give the vector expression of each of these forces.
   b. Deduce a 2nd order differential equation involving $\underline{u_m}$ as a function of *S*, *K*, *M*, $\underline{p_i}(O,t)$, $\underline{p_r}(O,t)$ and $\underline{p_t}(O,t)$.
   c. Why isn't there continuity of the overpressure between one side and the other side of the wall ?
   d. Why can we consider that the particle velocity is continuous between one side and the other side of the wall? Deduce a relation between $\underline{p_i}(O,t)$, $\underline{p_r}(O,t)$ and $\underline{p_t}(O,t)$. What is the relationship existing between $\frac{\partial \underline{u_m}}{\partial t}$ and the velocity of the particles for the transmitted wave $\underline{u_t}$? Deduce a relation between the transmitted overpressure $\underline{p_t}(O,t)$, $\underline{u_m}$, ω and Z.

3. Starting with the system of equations made of the second order equation in $\underline{u_m}$ and the two equations describing the continuity of the velocity of the particles, prove that the coefficient of transmission of the overpressure $\underline{t} = \frac{\underline{P_t}}{\underline{P_i}}$ is equal to :

$$\underline{t} = \frac{P_t}{P_i} = \frac{1}{1 + \frac{j}{2ZS}\left(M\omega - \frac{K}{\omega}\right)}$$

4. Represent schematically (with justification) the modulus of the transmission coefficient, $|\underline{t}|$, as a function of the angular frequency ω of the incident wave.
   a. To what type of filter does the wall correspond to? Express the resonant angular frequency ω_0.
   b. Numerical application : one considers a wall made of concrete of density $\rho = 2400\ kg/m^3$, of "spring constant" $K = 7.0\ 10^6 N/m$, width $l = 3.0m$, height $h = 2.5m$ and thickness $e = 10cm$. Calculate the value of ω_0.
   c. Knowing that the human ear can hear sounds of frequency 20 to 20 000 Hz, give a physical explanation why you hear mainly low pitched sounds (bass) when your neighbour listens to music.

Exercise 2 : Anti-radar device

Materials which give small reflection (and high absorption) of electromagnetic waves emitted by radars are known as *Radar Absorbent Materials* (RAM). They are used in stealth technology[1] since the years 1980.

Let's consider a metallic object producing a total reflection of a radar wave in normal incidence. To avoid this reflection the metal is covered by a layer of a RAM. This material has to fulfill two conditions: the first condition is that the incident wave should be totally transmitted at the air/RAM layer interface, the second condition is that the totally transmitted wave should be sufficiently damped within the RAM to avoid any reflection at the RAM/metal interface. We will assume in the following that the second condition is fulfilled and that there is subsequently no reflected wave at the RAM/metal interface.

The RAM is characterized by a permittivity ε and a permeability μ (both real values). This material will be considered here as an insulator (**negligible conductivity** γ), **uncharged, and with no surface current**. We will show that a proper choice of ε and μ, allows the total transmission of the incident wave at the air/RAM interface.

Let's consider a plane, harmonic, progressive incident wave, arriving on the interface air/RAM at z=0 under normal incidence. This plane wave will generally cause a reflected and a transmitted wave at the interface air/insulator (See Figure 3). We will use following notations :

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[1] « **Stealth technology** also termed **LO technology** (**low observable technology**) is a sub-discipline of military tactics and passive electronic countermeasures, which cover a range of techniques used with personnel, aircraft, ships, submarines, missiles and satellites to make them less visible (ideally invisible) to radar, infrared, sonar and other detection methods. It corresponds to military camouflage for these parts of the electromagnetic spectrum.» (https://en.wikipedia.org/wiki/Stealth_technology)

incident wave: $\overrightarrow{\underline{E_i}} = \underline{E_{io}}.\, e^{j(\omega t - \overrightarrow{k_i}.\vec{r})}\overrightarrow{e_x}$

reflected wave: $\overrightarrow{\underline{E_r}} = \underline{E_{ro}}.\, e^{j(\omega t - \overrightarrow{k_r}.\vec{r})}\overrightarrow{e_x}$

transmitted wave: $\overrightarrow{\underline{E_t}} = \underline{E_{to}}.\, e^{j(\omega t - \overrightarrow{k_t}.\vec{r})}\overrightarrow{e_x}$

where $\underline{E_{io}}$, $\underline{E_{ro}}$ and $\underline{E_{to}}$ may be complex numbers, $k_i = k_r = k = \frac{\omega}{c}$ and $k_t = \frac{\omega}{V}$, where V is the speed of the wave in the RAM. The permittivity and permeability of air will be taken equal to the values of vacuum and $c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$ is the speed of light in air.

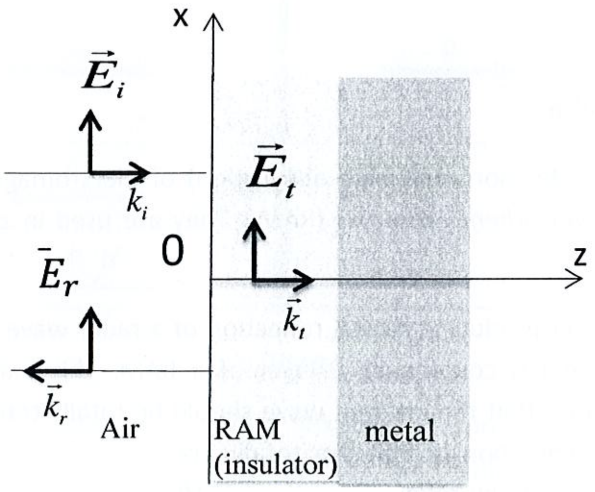

Figure 3

1. Determine the complex expressions for the magnetic fields $\overrightarrow{\underline{B_i}}$, $\overrightarrow{\underline{B_r}}$ and $\overrightarrow{\underline{B_t}}$.

2. Write the boundary conditions for the electric and magnetic fields at the interface z=0. Deduce the expressions of the coefficients for reflection $\underline{r_1} = \frac{\underline{E_{ro}}}{\underline{E_{io}}}$ and transmission $\underline{t_1} = \frac{\underline{E_{to}}}{\underline{E_{io}}}$ at the interface z=0, as a function of μ, μ_0, k and k_t.

3. <u>From Maxwell-Ampere equation,</u> find the expression of k_t as a function de ε, μ and ω.

4. Calculate $\underline{r_1}$ as a function of ε_r, μ_r.

5. What relation between ε_r and μ_r must be verified in the RAM in order to have $\underline{r_1} = 0$?

Boundary conditions :	$\overrightarrow{n_{1\to2}} \wedge \left(\frac{\overrightarrow{B_2}}{\mu_2} - \frac{\overrightarrow{B_1}}{\mu_1}\right) = \overrightarrow{k_s}$, $\quad \overrightarrow{n_{1\to2}}.\left(\overrightarrow{B_2} - \overrightarrow{B_1}\right) = 0$,
	$\overrightarrow{n_{1\to2}} \wedge \left(\overrightarrow{E_2} - \overrightarrow{E_1}\right) = \vec{0}$, $\quad \overrightarrow{n_{1\to2}}.\left(\varepsilon_2 \overrightarrow{E_2} - \varepsilon_1 \overrightarrow{E_1}\right) = \sigma$
Curl in Cartesian coordinates :	$\overrightarrow{rot}\, \vec{a} = \left(\frac{\partial a_z}{\partial y} - \frac{\partial a_y}{\partial z}\right)\overrightarrow{e_x} + \left(\frac{\partial a_x}{\partial z} - \frac{\partial a_z}{\partial x}\right)\overrightarrow{e_y} + \left(\frac{\partial a_y}{\partial x} - \frac{\partial a_x}{\partial y}\right)\overrightarrow{e_z}$