INSA | INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

FIMI Department
Second Year, 2018-2019

Physics: Written Test n°3

Friday January 25, 2018 — Duration: 3 hrs

Instructions:

Present your answers ***clearly and legibly.*** *Calculator* (any type but not internet connected) *and 3-pages formulary permitted.* ***Any illogical results, without comment, will be further penalized.***

Part 1

The MOS Transistor

Metal-Oxyde-Semiconductors (MOS) transistors represent the most fundamental component of micro-nanoelectronics. Millions of them are necessary to build an electronic memory, however they are all made according to the same structure shown in Fig.1. Their working principle is relatively simple: a conducting channel connecting two conducting areas called "source" and "drain" is created or suppressed in relation with the voltage applied on the grid. A voltage source V_{DS} being applied between source and drain as shown in Fig.1, for a given grid voltage V_0 the channel doesn't exist and consequently $i_{DS}=0$ whereas for another voltage V_1 the channel is created and consequently $i_{DS}\neq 0$. In this subject ***our goal is to study this structure when it is passing, i.e. when the channel between source and drain exists.***

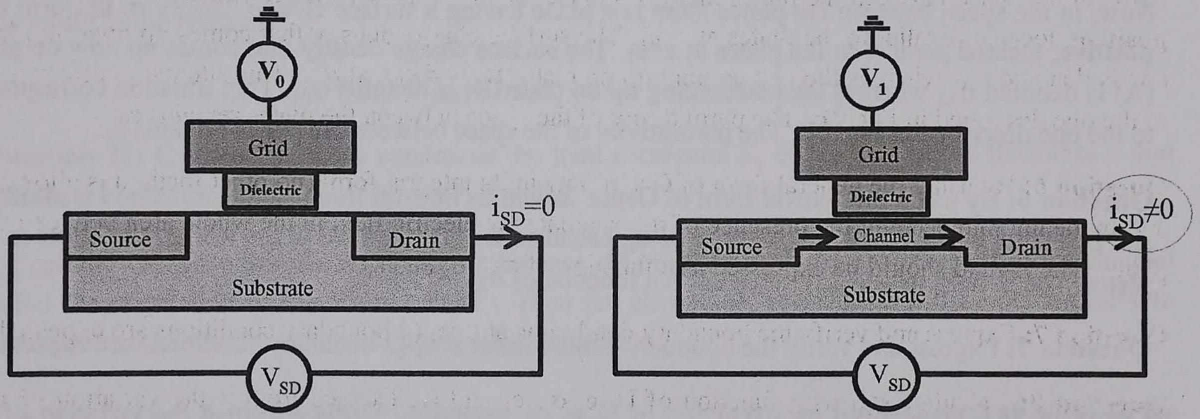

Figure 1: Working principle of the MOS transistor

The structure can be roughly modeled by a parallel-plate capacitor having **infinitely large plates**. This parallel-plate capacitor is made of two infinitely-large electrodes (A) and (B) which are first assumed to be made of metal. A potential difference U is maintained at their terminals as indicated in Fig. 2. Plate (A) represents the channel whereas plate (B) represents the grid. The surface charge density coming up in plate (A) is denoted σ_{A0} whereas the one coming up in plate (B) is denoted σ_{B0}. At first, the space between the plates is assumed to be filled by air of absolute permittivity ε_0: this situation is shown in Fig.2A. The whole device will be considered equivalent to a capacitor having plates infinitely large when considering symmetries and invariances. We will use the Cartesian coordinate system $(\vec{u_x}, \vec{u_y}, \vec{u_z})$ shown in Fig. 2A.

Question 1: Determine rigorously the direction of the electric field $\vec{E}$ existing anywhere in the space between the plates (for $0<z<e$), as well as the variables it depends on.

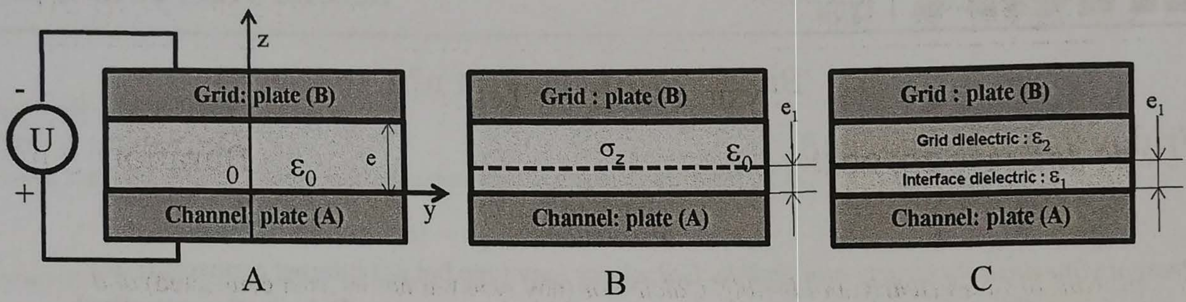

Figure 2: Modeling of the MOS transistor in conducting state by considering 3 different configurations (see detail in the text). Although the voltage source is present in the 3 configurations, it is only represented in the configuration A for simplicity

Question 2: Prove rigorously that $\sigma_{B0} = -\sigma_{A0}$.

Question 3: By use of Maxwell's equations in point form (only this method is allowed), calculate the electric field in the space between the plates defined by $0 < z < e$ as a function of the data of the exercise and in particular as a function of σ_{A0}.

Question 4: Calculate with a detailed demonstration the capacitance C_0 of such a transistor having a surface S as a function of ε_0, S and e.

Question 5: Numerical application: calculate C_0 if e=1.2 nm, $S=2.8\times10^{-14}m^2$, $\varepsilon_0=\frac{1}{36\pi}10^{-9}$ SI which corresponds to a distance between source and drain of 28 nm.

Now, in the space between the plates there is a plane having a surface charge density σ_z uniform and **positive**, located parallel to the plates at $z=e_1$. The surface charge density that comes up now on plate (A) is denoted σ_{A1} whereas the one coming up on plate (B) is denoted σ_{B1}. This situation corresponds to the one described in Fig. 2B. The permittivity of the space between the plates remains ε_0.

Question 6: By using the general form of Gauss' law in its integral form (no other method is allowed) and by taking into account the presence of σ_z, calculate the electric field in the whole area between the plates. The E-field should be expressed as a function of σ_{A1}, σ_z and ε_0.

Question 7: Express and verify the boundary conditions at $z=e_1$ (2 boundary conditions are expected).

Question 8: Calculate σ_{A1} as a function of U, e, σ_z, e_1 and ε_0. Deduce from it the variation of the surface charge density in the plates, $\Delta\sigma = \sigma_{A1} - \sigma_{A0}$ due to the presence of the plane charged with σ_z.

Question 9: Determine the expression of the potential V(z) between z=0 and z=e and plot V(z) schematically. To do that, assume that $V(0) = U > 0$ and that $V(e) = 0$.

Question 10: Bonus: what will be the influence of the presence of a charged plane located in the space between the plates (like σ_z) on the current i_{SD}?

In real devices, the space between the plates is not filled with air but with a dielectric. Currently this dielectric is a hafnium oxide of absolute permittivity ε_2 with possibly traces of nitrogen or silicon: HfO_2, $HfSiO_2$ or HfSiON. The growth of Hafnium oxide on silicon induces several defects at the interface between the two materials. Such defects are detrimental to the performances of the transistor. To reduce the defects, we insert between the silicon and the hafnium oxide a layer of SiO_2 of absolute permittivity ε_1. The final structure, represented in Fig. 2C, still has a total thickness e. Subsequently, the thickness of the SiO_2 layer being denoted e_1 and that of the hafnium oxide e_2, we have $e=e_1+e_2$. In the rest of the exercise, as the charged plane σ_z doesn't exist anymore we will denote σ_{A2} and σ_{B2} the surface charge densities coming up on the plates (A) and (B).

Question 11: Determine the expression of the new capacitance C_1 of this structure. Justify imperatively your answer. Compare with the capacitance C_0 knowing that the smallest among the permittivities ε_1 and ε_2 is approximately equal to four times ε_0.

The geometry of the channel in working conditions is represented in Fig. 3. When the voltage V_{SD} is applied between source and drain, an electric field $\overrightarrow{E_{SD}}$ assumed to be ***uniform*** *is created in the direction (Oy):*

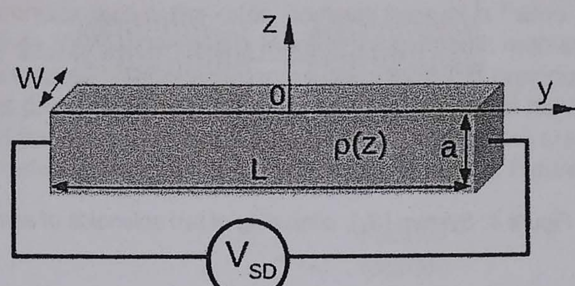

Figure 3: Channel of the MOS transistor polarized by a voltage V_{SD}

$$\overrightarrow{E_{SD}} = E_{SDy}.\vec{u}_y, \text{ with } E_{SDy}=cst$$

The resistivity of the channel in working conditions is not perfectly uniform. It depends on the depth according to the relation:

$$\rho(z) = \rho_0 - \alpha z \text{ if } -a < z < 0 \text{ with } \alpha > 0$$

Question 12: Calculate in these conditions the total resistance R_c of the channel of thickness a and volume $\tau = aWL$ as a function of a, ρ_0, W, L and α.

Indications: calculate first the elementary resistance dR of an appropriately chosen part of the channel (mind the orientation of the electric field!). Then the elementary resistances will be put together to build up the total resistance. Please justify all calculations.

If $\rho_0 >> \alpha a$ then the resistance R_C becomes: $R_C = \rho_0.\frac{L}{Wa}$ (well-known formula corresponding to the case of a rod, homogeneous and with constant cross-section)

Question 13: Numerical application: In the framework of this approximation calculate R_C if $\rho_0=10^{-4}$ Ω.cm, $a=2$ nm, $W=1$ µm, $L=28$ nm.

Part II : Synchronous motors

A synchronous motor consists of a stator and a rotor. The stator produces the rotating magnetic field which creates the torque that rotates the rotor. We will first study the principle of the stator.

The stator consists of two identical systems (S_1) and (S_2); one of them, shown in Figure 4, consists of two solenoids of same axis, connected in series so that the current flowing through them has the same orientation. In the case of the system (S_1), the axis of the solenoids is denoted Ox.

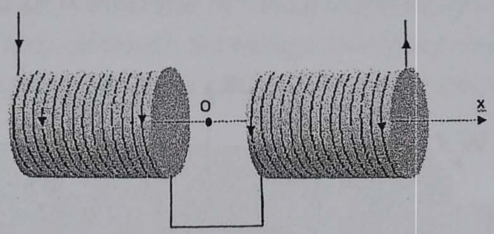

Figure 4: System (S_1), consisting of two solenoids of same axis.

Question 1 : Give (and justify) the direction and orientation of the magnetic field $\vec{B}$, produced by the system (S_1) at point O. (We will neglect in the study of the topography of the $\vec{B}$ field the presence of the cables connecting the 2 solenoids).

At point O, equidistant from the two solenoids where a current i is flowing, it will be admitted that the norm of the $\vec{B}$ field is $\left\|\vec{B}\right\| = K.i$.

We now place two identical systems (S_1) and (S_2), perpendicular to each other. Each system has an equivalent resistance R and a total inductance L and is part of an electrical circuit shown in Figure 5 (where R is not shown, and R_0 represents a resistance different from R). The voltage source delivers a sinusoidal voltage $u(t) = U_0\, cos(\omega_0 t)$. We will use a Cartesian coordinate system $(\vec{u}_x, \vec{u}_y, \vec{u}_z)$ defined on Figure 5.

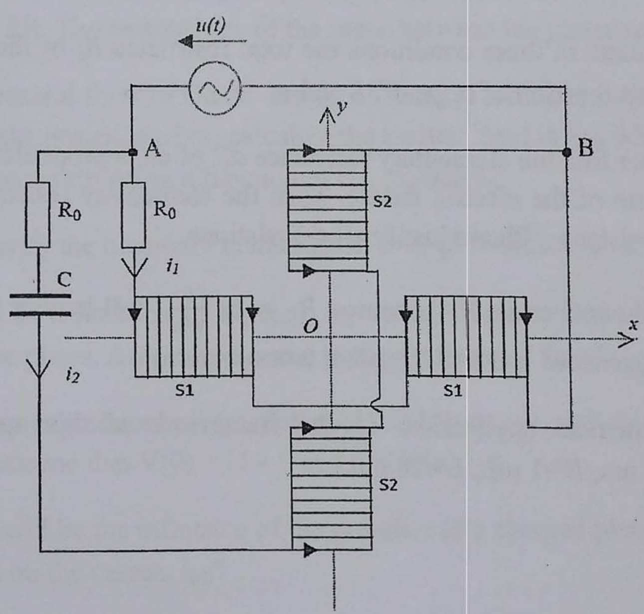

Figure 5: Sketch of the stator constiting of two systems (S_1) and (S_2) perpendicular to each other.

The current i_1 flowing through S_1 is of the form $i_1(t) = I_1\, cos(\omega_0 t - \varphi_1)$, the current i_2 flowing through S_2 is of the form $i_2(t) = I_2\, cos(\omega_0 t - \varphi_2)$.

Question 2 : Express I_1, I_2, φ_1 and φ_2 as a function of U_0, R, R_0, L, C and ω_0 (a simplified circuit diagram of the device will be drawn beforehand and any mutual induction between the coils will be neglected).

Question 3 : R, L, and ω_0 being fixed, what value should be given to C to have $I_1 = I_2$?

One could further demonstrate that $\varphi_2 - \varphi_1 = \frac{\pi}{2}$ when $R_0 = L\omega_0 - R$.

The following questions do not need to have answered the previous questions.

Question 4 : Considering the previous conditions fulfilled ($\varphi_2 - \varphi_1 = \frac{\pi}{2}$, $I_1 = I_2$), give the expression of the total field $\vec{B}_T$ at point O as a function of K, ω_0, φ_1 and I_1.

Question 5 : Justify the name of rotating field for the field $\vec{B}_T$.

The moving part of the synchronous motor, the rotor, consists here of a frame around which a wire is wound, building up a current loop (of surface S) equivalent to a magnet, of magnetic moment $\vec{M}$. We will first consider the current in the current loop constant, and so will be the norm M_0 of $\vec{M}$. The previous $\vec{B}_T$ field, which rotates at the speed ω_0 (same angular speed as priviously) around the Oz axis, is supposed here uniform in the whole region of the rotor. It is further assumed that the rotor, and thus $\vec{M}$, also rotates around the Oz axis with an angular velocity ω. The vectors $\vec{B}_T$ and $\vec{M}$ thus form between them an angle $\theta(t)$ at any time (see Figure 6).

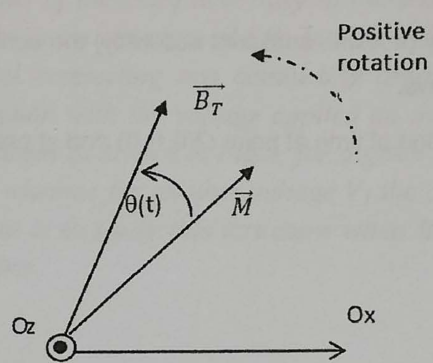

Figure 6: Relative positions of the $\vec{B}_T$ field and the magnetic moment $\vec{M}$

Question 6 : Knowing that $\theta(0) = \theta_0$, express $\theta(t)$ as a function of ω, ω_0 and θ_0.

The expression of the torque exerted by the field $\vec{B}_T$ on the rotor is given: $\vec{\Gamma} = \vec{M} \wedge \vec{B}$.

Question 7 : Give the expression of $\left\|\vec{\Gamma}\right\|$ as a function of M, B_T, ω, ω_0 and θ_0. Why can the synchronous motor only work for $\omega = \omega_0$?

Several phenomena prevent this condition ($\omega = \omega_0$) from being easily fulfilled. In particular, since the angle varies, which is inevitably the case in the engine start phase, the flux of the magnetic field $\vec{B}_T$ through the frame also varies.

Question 8 : Express the variation of the flux as a function of $\theta(t)$, $\dot{\theta}(t) = \frac{d\theta}{dt}$, B_T, and S (we will still consider that the rotor is subjected to a uniform field over all its whole section S).

Question 9 : What will then happen in the rotor current loop? Can we consider, as we did previously, that the current flowing in the rotor is constant? Why ?

Question 10 : What can be said about the magnetic moment $\vec{M}$? Why does the rotor also affect the stator?

Question 11 : Make a sketch of the electrical circuit equivalent to the rotor knowing that the latter is powered by a generator delivering a voltage $u_r(t)$ and that it has a resistance R_r and a self-inductance L_r. Write the electrical equation of this circuit as the function of the data of the text.

To conclude, we see that the synchronous motor is not as simple as it seems and requires a fine control of the current and/or the voltage creating the rotating field and the magnetic moment.

Part III : Waves

A wave carrying a vector signal $\vec{S}$ is represented by the following equation (with S_0, and δ positive constants) :

$$\vec{S}(x,y,z,t) = S_y\,\vec{u}_y = S_0\,e^{-\delta|x|}e^{-\delta|z|}cos(\omega t - ky)\,\vec{u}_y$$

Answer the following questions, whose purpose is to describe this wave, justifying imperatively each answer :

Question 1 : What is the direction of propagation of the wave?

Question 2 : Is this wave uniform?

Question 3 : What is the direction of polarization of the wave?

Question 4 : Is this wave transversal or longitudinal?

The frequency of the wave is $f = 915\ MHz$, its wave number $k = 19.8\ rad.m^{-1}$. We also give: $\delta = 8.45\ m^{-1}$

Question 5 : Calculate the values of the wavelength and of the phase velocity.

Question 6 : Represent schematically (but with sufficient accuracy) the component S_y of $\vec{S}$ along the (Oz) axis, for $t = \frac{T}{2}$, where T is the period of the wave.

Question 7 : Represent S_y as a function of time at point $O(0;0;0)$ and at point $M(0;7.9\ cm;0)$.