INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

FIMI 2nd Year **Year 2018/2019**

Physics : Exam n° 1

Monday 15th October 2018 **Duration : 1 h 30**

Tentative Marking Scheme: exercise I : 13 points, exercise II : 7 points. Calculator, documents and phones are forbidden.

The subject comprises two independent exercises.

Errors in units may be penalized.

Numerical values of physical constants: $\varepsilon_0 = \frac{1}{36\pi 10^9}$ SI units, $\mu_0 = 4\pi 10^{-7}$ SI units

Divergence operator in Cartesian coordinates:

$$div(\vec{X}) = \frac{\partial(X_x)}{\partial x} + \frac{\partial(X_y)}{\partial y} + \frac{\partial(X_z)}{\partial z}$$

Gradient and divergence operators in spherical coordinates:

$$\overrightarrow{grad}(V) = \frac{\partial V}{\partial r}\vec{u}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\vec{u}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\vec{u}_\varphi$$

$$div(\vec{X}) = \frac{1}{r^2\sin\theta}\left(\frac{\partial(r^2\sin\theta X_r)}{\partial r} + \frac{\partial(r\sin\theta X_\theta)}{\partial \theta} + \frac{\partial(rX_\varphi)}{\partial \varphi}\right)$$

Exercise I – Model of a thunderstorm cloud

When the weather is fine, water vapor goes up from the earth to the atmosphere to build up a cloud. Along with the water vapor there are also electrons that have been teared off from the ground. In our simplified model:

*- The earth is plane and fills the whole space located in the region $z \leq 0$. The ground (located at z =0) bears a **uniform and positive surface charge density σ**;*

*- A cloud is represented by an object which is infinite in the x and y directions and which is located between the altitudes $z = H$ and $z = H+h$ (where h is the thickness of the cloud), and bears an **inhomogeneous volume charge density $\rho(z) = a.(z-H-h)$,** where a is a constant.*

1) What is the unit of a? The charges in the cloud being negative, what is the sign of a?

2) Analyze rigorously the symmetries and invariances of the charge distribution with the help of a sketch and deduce from it the topography of the electric field $\vec{E}$.

3) We assume that $\vec{E} = \vec{0}$ for z > H+h. Deduce the electric field in the cloud (H < z < H+h) by using a local relation (point-form equation). (Note: the polynomial obtained is the square of the sum or difference of two terms).

4) Calculate the electric field below the cloud (0 < z < H), once again by using a local relation as well as boundary conditions if necessary.

5) Draw the graph of the norm of $\vec{E}$ as a function of z for $0 < z < H+h$.

6) We assume that $\vec{E} = \vec{0}$ for $z < 0$. By applying rigorously Gauss' law in its integral form, deduce the charge Q on a surface S of the ground as a function of *a*, *h* and *S*.

7) What is the charge Q' in a cylinder of cloud of axis Oz, section S and located between H and H + h? Comment.

8) Numerical application: H = 0.5 km, h = 1 km. Let's assume that the maximum value for the norm of the electric field is the value triggering lightning in humid air, i.e. approximately 10^6 V/m. Give the order of magnitude for the charge Q with S = 100 m^2.

Exercise II – Vector operators and Maxwell equations

The Japanese nuclear physicist Yukawa suggested in 1935 a simple formula to describe (in spherical coordinates) the electric field created by an hydrogen atom in its fundamental state 1s, modelled by a point charge +e in the center (the proton) surrounded by a "cloud" of charge (with spherical symmetry) corresponding to the electron –e :

$\vec{E}(\mathrm{r}) = \frac{e}{4\pi\varepsilon_0 r^2}\left(1+\frac{r}{a}\right) exp\left(-\frac{r}{a}\right)\vec{u_r}$, where a is a positive constant.

A- Without calculations, what is the value of $\overrightarrow{rot}(\vec{E})$ and why?

B- Give the mathematical equivalents of the function $\vec{E}(\mathrm{r})$ for r << a and for r >> a.

C- From the electric field (not approximated) deduce the charge density ρ(r) of the electronic charges.

D- From the electric field (not approximated) deduce the Yukawa potential V(r), assuming it tends toward 0 when r is very big with respect to a.