Physics: Exam n° 2

Monday 3rd December 2018 **Duration: 1h30**

Tentative marking scheme: exercise 1: 13 points, exercise 2: 7 points.
One handwritten sheet, written on both sides and calculator permitted

Exercise 1 – Sizing of a solenoid (13 pts)

1. Consider the electric circuit of Figure 1 which is constituted of a capacitor C in series with a solenoid (L, R) connected at the terminals of a voltage source. Note that R represents the resistance of the wire of the solenoid and L its inductance.

Assume that the voltage delivered by the generator is sinusoidal of angular frequency ω_0.

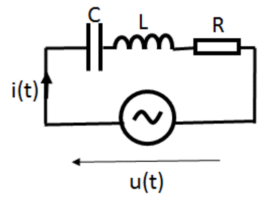

Figure 1

1.1 Calculate the complex impedance of this circuit, put it in the form $a + jb$.
1.2 Deduce that the current i(t) and the voltage u(t) are in-phase when $LC\omega_0^2 = 1$

We wish to make such a solenoid (L, R) by using a copper wire which surface is insulated (for simplicity, we will neglect this insulator surrounding the wire in the rest of the exercise).

2. The copper wire is considered to be equivalent to a conductor of length ℓ, diameter d and conductivity γ.

2.1 It is submitted to a potential difference U. Give the equation involving an integral which proves that an electric field $\vec{E}$ exists in the conductor.

2.2 Prove with the help of another (local) equation, that there is a volume current density $\vec{j}$ in the conductor. Deduce that $\vec{E}$ is collinear to $\vec{j}$ and that $\vec{j}$ is uniform if it is the case for $\vec{E}$.

2.3 We will admit that when a DC current is flowing through the conductor, the volume current density is uniform throughout the conductor and that it is of the type $\vec{j} = J_0\vec{u_z}$ where J_0 is a positive constant (figure 2). Give the literal expression of the current I flowing through a cross-section S of the conductor. To do so, express the elementary current dI flowing through a surface element dS and make an illustrating sketch with all useful elements.

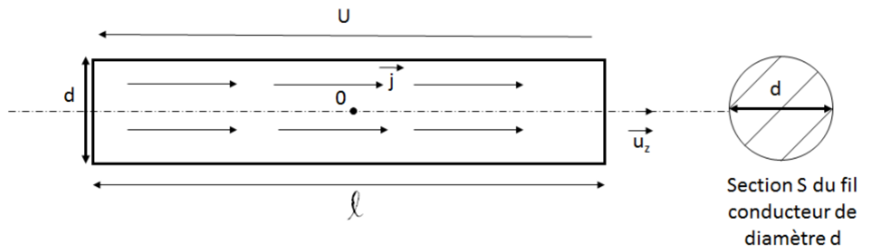

Figure 2

2.4 Express the potential difference U at the terminals of the conductor as a function of J_0 and then as a function of I and deduce R, the resistance of the wire as a function of ℓ, d and γ.

3. We wind the wire **over its complete length ℓ** around a thin support having the same properties as vacuum in order to fabricate a solenoid of diameter D **We assume that the turns (or loops) join each other and are perfectly circular.**

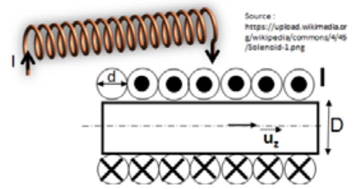

Figure 3

3.1 Assuming the solenoid to be infinitely long, determine the topography of the magnetic field $\vec{B}$ created by this solenoid when it is crossed by the current I.

3.2 Prove that $\vec{B}$ is uniform inside the solenoid. It is recalled that its modulus is equal to $\mu_0 nI$ where n represents the number of turns per unit length.

3.3 Knowing that the wire has a diameter d, give for this wire the literal expression of the number of turns n per unit length as a function of d. (*Hint : this is equivalent to calculating, as a function of d, the number of turns necessary to make a solenoid of 1 m long*).

3.4 Give the literal expression (as a function of D and ℓ) of the number of turns N that can be done with this wire of length ℓ.

3.5 Give the literal expression (as a function of μ_0, d, ℓ and D) of the inductance L of this solenoid.

4. The electrical circuit works in the conditions of question 1.2. Reminder : $\mu_0 = 4\pi 10^{-7} Hm^{-1}$.

Circuit characteristics			Copper wire characteristics	
C (µF)	R (mΩ)	ω_0 (rad.s^{-1})	d (mm)	γ (S.m^{-1})
180	100	6283.4	1.5	58.7 *10^6

4.1 From the expression of R (question 2.4), deduce the expression of ℓ, (length of the cable) as a function of the data in the above table and do the numerical application.
4.2 From the expression of L (question 3.5) and the condition on L, C, ω_0 (question 1.2), deduce the expression of D as a function of the data in the above table and do the numerical application.
4.3 Deduce from the previous questions the numerical value for N.

5. We would like to work for an angular frequency 10 times smaller than ω_0.

5.1 By which factor should we multiply the inductance L in order to get the same behavior as previously for the circuit at this new angular frequency?

5.2 Taking into account the fact that the support of the solenoid is hollow, suggest a technological solution to get this new value for L without having to create a new solenoid.

Exercice 2 – Energy transfer (harvesting) by use of static induction (7pts)

We would like to evaluate the power that can be retrieved by transfer (harvesting) of magnetic energy. For this, we will study a very simple set-up: an infinite wire (on the zz' axis) carrying a current $i_f(t) = I_f\sqrt{2}\,\cos(\omega t)$ is placed at a distance d from a closed circuit, square-shaped, of side a (see figure 4) with a resistance R=10Ω (representing for instance a mobile phone that you want to charge).
The self-induction is neglected in questions 1 to 3.

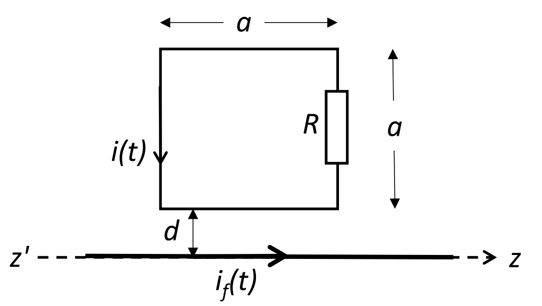

Figure 4

1. Determine the magnetic field $\vec{B}$ created by the infinite wire. You will consider that even if the quantities vary with time, they vary slowly enough to be able to use the magneto-static methodology. Indicate what approximation is implicitly done when doing so.
2. Express the electromotive force $e(t)$ induced in the square circuit in the form $E\sqrt{2}\,\sin(\omega t)$. Give the expression of E as a function of I_f, a, d, ω and any other necessary constants. Deduce the intensity in the circuit $i(t)$ in the form $I\sqrt{2}\,\sin(\omega t)$. Give the expression of I as a function of I_f, R, a, d, ω and any other necessary constants.

We recall that in sinusoidal regime the average power dissipated in a resistance is: $P = RI^2$ (with I: the RMS intensity).

3. Numerical application: I_f =10A, f =50Hz, d =10mm, a =50mm, $\mu_0 = 4\pi 10^{-7}$Hm^{-1}.
Calculate the power P dissipated in the resistance (transferred by use of static induction). Comment. How could you increase this power (give at least 2 suggestions)?

4. We consider now that the square circuit has a self inductance L =100mH. The new intensity is: $i'(t) = I'\sqrt{2}\,\sin(\omega t)$. Give the expression of I' as a function of E, R, L, ω. What would be the new power P' dissipated in the resistance? Do the numerical application and comment.