INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

PC Second Year

Physics: Test n°3 – Semester 2

Monday May 6th | Duration: 1h30

***Indicative marking scheme*: exercise 1 on 10 points, exercise 2 on 10 points**
The subject is made of 2 independent exercises
Authorized document: one double-sided synthesis sheet, original and handwritten
Authorized calculator: any non internet-connected type
No mobile phone allowed

Exercise 1: Interferences

We consider the following set-up placed in a medium of index $n_0=1$: a monochromatic point source S illuminates a thin converging lens of focal length f.
The source is located at the distance: $\overline{SO} = d = 3f/2$ from the lens on the Ox axis. A screen is placed perpendicularly to the Ox axis at a distance x (which may be adjusted) from the lens.

As described in Figure 1 (to be handed in with your manuscript) the lens is then cut in its middle along the plane (x, z) which is the horizontal plane in two half-lenses L_1 and L_2, of height h and separated by the distance $e = O_1O_2$ (O being in the middle of O_1O_2). These two half lenses (which of course remain of focal distance f) are slightly inclined with respect to the vertical axis. The optical axes of the two half-lenses correspond to the two rays indicated with dotted lines in Figure 1 and they cut the half-lenses at points O_1 and O_2 respectively.

The area separating the two half-lenses is made opaque by positioning a screen of width *e* corresponding to the distance O_1O_2. In doing so, any direct illumination of the screen by the source S is completely avoided.

1. We denote S_1 and S_2 the images of the source S by the two half-lenses respectively. These images build secondary sources. As the angle of inclination between the half-lenses and the y-direction is small, we will consider that $SO_1 = SO_2 \approx SO = d$.
   a. *Express OS_1 and OS_2 as a function of d.*
   To do this calculation it is recalled that for a thin lens of focal distance f centered at point I, the image A' of an object A is given by the thin lens equation:
   $$\frac{1}{\overline{IA'}} - \frac{1}{\overline{IA}} = \frac{1}{f}$$ **with $\overline{IA}$ algebraic distance (> 0 if along increasing x, < 0 otherwise)**
   b. *Report in Figure 1 the positions of S_1 and S_2.*
   c. *Express the distance between the two secondary sources S_1 and S_2. This distance is denoted s. First express s as a function of e, d and SS_1 and then express s only as a function of e.*
   As the angle of inclination of the two half-lenses with the y-axis is small, we will approximate in the following that the abscissa of S_1 and S_2, called d', is equivalent to the length OS_1.
   In the rest of the exercise use the variables s and d' even if you haven't answered question 1.

2. From the previous question, it comes out that this set-up creates two sources S_1 and S_2, separated by the distance s and located at the distance d' from O.
   a. *Describe what is observed on the screen located at the distance x = OR.*
   b. *Complete Figure 1* ***(to be handed in with your manuscript)*** *by drawing the 2 beams of light passing through the two half-lenses. To do that first complete the paths of the 2 rays indicated by dotted lines in Figure 1 which are incomplete (the rays crossing O_1 and O_2 being*

already complete). You will also clearly highlight the interference area (for example by hatching it).

3. *Give the literal expression of the interfringe distance i as a function of x, d', s and λ. A formula seen in the lecture or tutorial can be re-used without demonstrating it.*

4. The monochromatic source located at point S is now replaced by a **white light source**. *Describe the changes observed on the screen with respect to the previous case.* Assume that we place a grating spectroscope at a point M of the screen located in the interference area. Point M is located at the distance y = 1 mm from point R. *Describe the light spectrum observed and give the number of black fringes observed in the spectrum knowing that the distance between the screen and the two secondary sources is 0.5 m and that s = 3mm (note that the values of the corresponding wavelengths are not asked for).*

Exercise 2: Fraunhofer diffraction by a small angle prism

A plane, progressive, uniform monochromatic light wave of wavelength λ_v in vacuum arrives with normal incidence on one side of a prism (figure n°2). This wave has been created by a point source S at infinity on the (Oz) axis.

The prism, made of glass of refraction index n, has a very small angle α and is placed in a media of index n_0. It's height OA in the direction (Ox), called *a*, is small compared to its length *b* in the direction (Oy) : $a << b$

The small height *a* of the prism prompts us to study the diffraction of the wave by the prism. Point P is at infinity, <u>in the direction of observation</u> θ (Figure n°2)

a. *Show on Figure 2* ***(that you will return with your manuscript)*** *the optical path difference at the point P (at infinity) between the ray passing through O and the ray passing through M.*

b. *Express this optical path difference, called δ_{MP-OP} as a function of x where x=OM, of the indices, and of the angles α et θ. To simplify the calculation, you'll consider α very small; you'll not make any approximation on θ for now.*
Hint: several methods can be used. One method consists in calculating MN, then ON. Then you will recall that cos(π/2-ϕ)=sinϕ and that sin(a+b)=sin(a)cos(b)+sin(b)cos(a)... Another method consists in calculating MN, then ON' and N'N...

c. *Give the expression of the integral leading to the calculation of the complex amplitude of the total wave reaching point P at infinity. You will make δ_{MP-OP} appear in this expression, as well as all the terms which you will consider necessary and <u>which you will explain clearly.</u>*

d. In this integral δ_{MP-OP} *can be expressed as a product: $x.f(\alpha,\theta)$ with f(α, θ) being a function of α and θ.* Starting with this integral, show that the intensity at point P is : .

$$I = I_{max} \times \text{sinc}^2\left(\pi \frac{a}{\lambda_v} f(\alpha,\theta)\right)$$
. (sinc is the "sinus cardinal function" : sinc(X)=sin(X)/X)

e. In conclusion, I is equal to $I_{max} \times \text{sinc}^2\left(\pi \frac{a}{\lambda_v}\left[n_0 \sin\theta - (n - n_0\cos\theta)\alpha\right]\right)$. Since the angles θ are small, we can approximate this expression by keeping only the terms of order 1. *Give the new expression of I. Draw the approximate shape of the curve I(θ), making the relevant quantities appear on the figure.*

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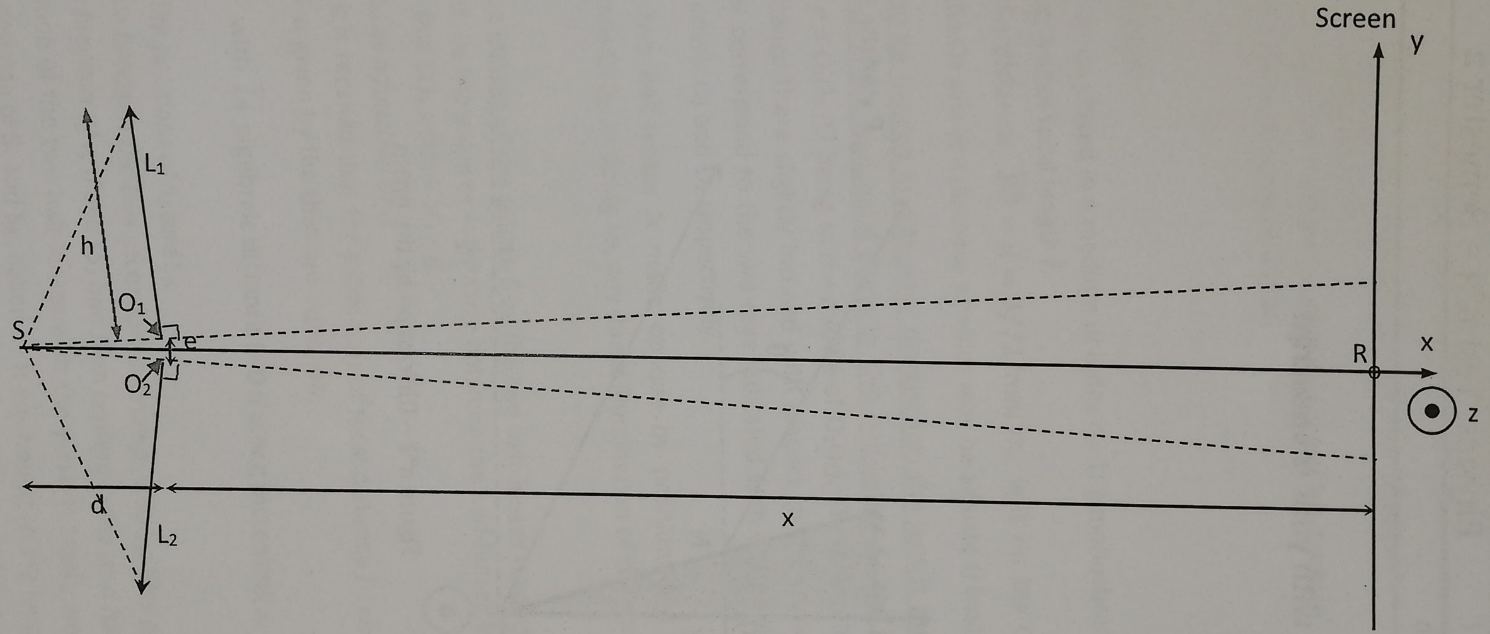

Figure 1 : Interferometric device

(figure made with a distance e between the two half-lenses increased with respect to the other distances to facilitate the drawing of the rays)

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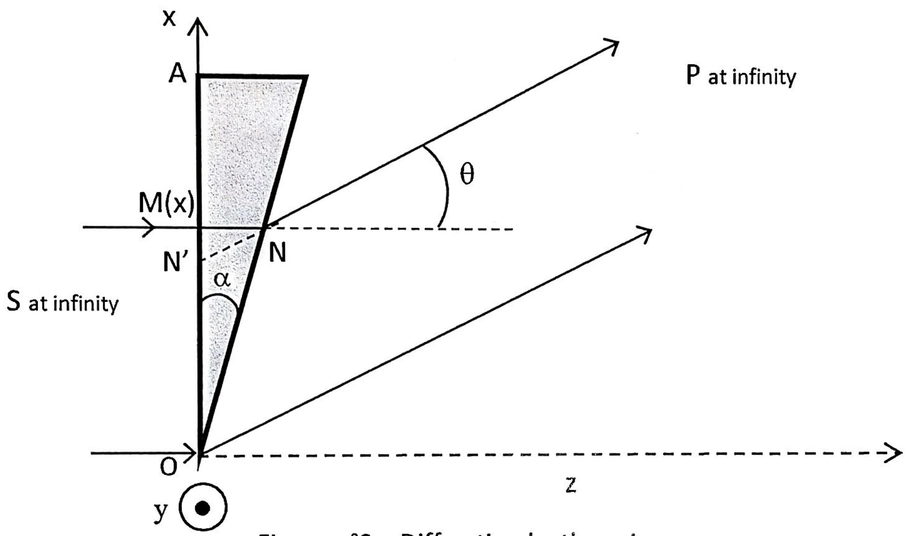

Figure n°2 – Diffraction by the prism