Physics: Test n°2

Monday 2 December **Duration: 1h30**

Indicative marking scheme*: *exercise 1 is worth 10 points, exercise 2 is worth 10 points.

Maxwell's equations are recalled in local form in a medium (insulator or conductor) having the same dielectric and magnetic properties as that of vacuum ($\varepsilon = \varepsilon_0$ and $\mu = \mu_0$):

$$\overrightarrow{rot}(\vec{B}) = \mu_0 \vec{j} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \; ; \quad div(\vec{B}) = 0 \; ; \quad \overrightarrow{rot}(\vec{E}) = -\frac{\partial \vec{B}}{\partial t} \; ; \quad div(\vec{E}) = \frac{\rho}{\varepsilon_0}$$

Moreover, we recall the expressions of the rotational and of the divergence in cylindrical coordinates applying to a vector $\vec{X} = X_r\,\vec{u_r} + X_\theta\,\vec{u_\theta} + X_z\,\vec{u_z}$:

$$\overrightarrow{rot}(\vec{X}) = \frac{1}{r}\left(\frac{\partial X_z}{\partial \theta} - \frac{\partial (rX_\theta)}{\partial z}\right)\vec{u_r} + \left(\frac{\partial X_r}{\partial z} - \frac{\partial X_z}{\partial r}\right)\vec{u_\theta} + \frac{1}{r}\left(\frac{\partial (rX_\theta)}{\partial r} - \frac{\partial X_r}{\partial \theta}\right)\vec{u_z}$$

$$div(\vec{X}) = \frac{1}{r}\left(\frac{\partial (rX_r)}{\partial r} + \frac{\partial (X_\theta)}{\partial \theta} + \frac{\partial (rX_z)}{\partial z}\right)$$

We also recall the expression of the gradient of a scalar function in cylindrical coordinates $F(r,\theta,z)$ *:*

$$\overrightarrow{grad}(F) = \frac{\partial F}{\partial r}\vec{u_r} + \frac{1}{r}\frac{\partial F}{\partial \theta}\vec{u_\theta} + \frac{\partial F}{\partial z}\vec{u_z}$$

Exercise 1: Digital Micromirror Devices (DMD)

Some video projectors **use Digital Micromirror Devices** *(DMD). A cell of DMD (which represents one pixel) is constituted in particular of a microelectromechanical system made of an aluminum micromirror fixed on a rotation axis (Δ) which is bound to a metal plate (Figure 1.a). The whole (micromirror + plate) acts as a seesaw: it can rotate around the axis (Δ) of an angle α (α can reach ± 10°) with respect to the horizontal position. This rotation occurs under the action of electrostatic forces.*

It is this movement of rotation that enables the Al micromirror to reflect light towards a projector. In the following, we will admit that each device (plate + electrode) is equivalent to a capacitor whose plates build an angle α called dihedral angle (Figure 1.b).

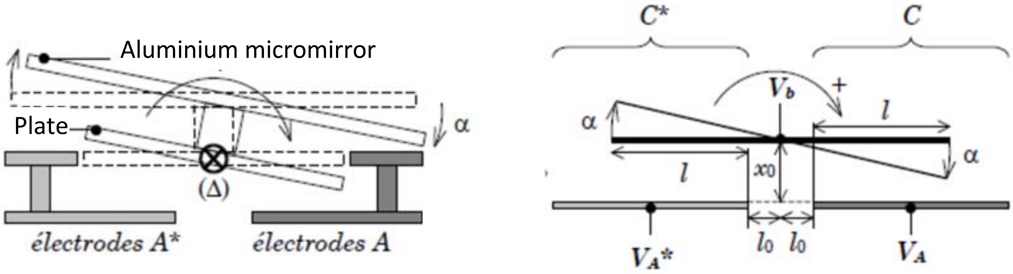


Figure 1 : a) Electromechanical system made of an aluminium micromirror and a metal plate (acting as a seesaw) ; b) Equivalent system made of two capacitors, each of them having plates making an angle α (also called dihedral[1] capacitors).

To assess the time response of the cell (with respect to the clock signal set by the video signal), it is necessary to determine the capacitance of these capacitors.

[1] A dihedral angle is the angle built by the intersection of two planes

In this context we will study the working principle of the dihedral capacitor shown in Fig. 2*: the capacitor consists in two plates located at the same distance R_1 from an axis (Oz) and forming an angle α. The two plates are made of a perfect conducting material, rectangular and they have the same surfaces $S = h\,(R_2 - R_1)$. The length h of the plates along the direction z of unit vector $\vec{u}_z$ is assumed to be very large. A positive potential difference $U = V_1 - V_2$ is maintained between the two plates: plate (1) is at the potential V_1 and plate (2) at the potential V_2. We denote (r, θ, z) the coordinates of an arbitrary point M located in the space between the plates. Edge effects are neglected.*

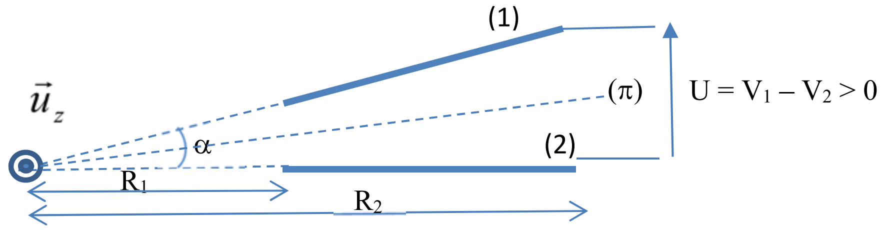


Figure 2: Sketch of the dihedral capacitor studied.

1) Through the study of the symmetries of the charge distribution, justify that the plane (π) which cuts the angle α in two equal parts (it is a bisector plane) builds up an equipotential surface.

In the following we will consider that all portions of planes containing the z axis and located in the space between the capacitor plates are equipotential surfaces.

2) What is then the direction of the electrostatic field? Justify briefly. Represent some field lines on a sketch and indicate their orientation.

3) With the help of one of Maxwell's equation in point form, explain why the modulus of $\vec{E}$ doesn't vary along a field line.

4) Using the relation between $\vec{E}$ and U, prove that $\vec{E}$ can be expressed in the form: $\vec{E} = \frac{K}{r}\vec{u}$ where K is a constant to be given as a function of the appropriate parameters and $\vec{u}$ is a unit vector to express[2].

Deduce the surface charge density $\sigma_1(r)$ et $\sigma_2(r)$ on the two plates of the capacitor. Comment on the expressions obtained and justify them briefly.

5) Give the literal expression of the capacitance C of this capacitor.

Numerical calculation: First calculate the capacitance C of a single capacitor, then the equivalent capacitance for the 1024 x 768 capacitors of a video projection system. The capacitors are considered as arranged in parallel. Use the following values for the calculation:

$R_1 = 2.0\ \mu m$, $R_2 = 7.0\ \mu m$, $h = 20.0\ \mu m$, $\alpha = 10°$ et $\varepsilon_0 = 8{,}9\ .10^{-12}$ SI

6) *The complete system (all the DMD -or pixels- and connecting wires) is equivalent to a capacitor of capacitance C' = 2 nF in series with a resistance R (known as bridge resistance).* What is then

[2] This expression of $\vec{E}$ can be used in the rest of the exercise even if you couldn't demonstrate it.

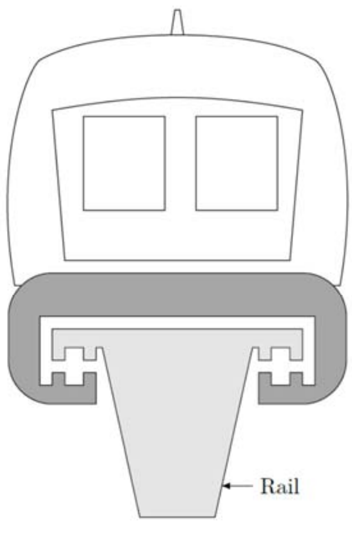
the maximal bridge resistance for the equivalent electrical system allowing to follow a video signal whose period is of 20 μs? Justify your answer.

Exercise 2: Working principle of a magnetic levitation train

A magnetic levitation train (maglev), contrary to standard trains, uses magnetic forces to levitate over the rails, allowing it to never be in contact with them. This feature removes the contact friction, which in turn allows very high speeds to be attained.
In this exercise we will focus on the electromagnetic levitation principle in which a train levitates by magnetic attraction using magnets. The 30 km line between Shanghai and the Pudong International Airport is, since 2004, the only line in commercial operation today. The journey takes under 8 minutes at an average speed of 245 km/h. Figure 3 shows the section of a Transrapid car on its rail as well as the detailed view of its levitation system. It consists of an electromagnet whose magnetic circuit is composed of:

- *A portion of the rail (1),* ***motionless****, made of a soft ferromagnetic material with relative permeability μ_r;*
- *A portion (2), bound to the train car, made from the same soft ferromagnetic material, on which an N-turn coil is wound. This portion is free to move along the $(O, \vec{u}_z)$ axis. The coil is powered with a constant current i_0.*

Both portions are separated by an air gap of width z (which is likely to vary). Let (C) be a magnetic field line, as seen in figure 3. The surface S of the cross-section of the ferromagnetic material (perpendicular to (C)) is assumed to be the same in portions (1) and (2) throughout the whole magnetic circuit.



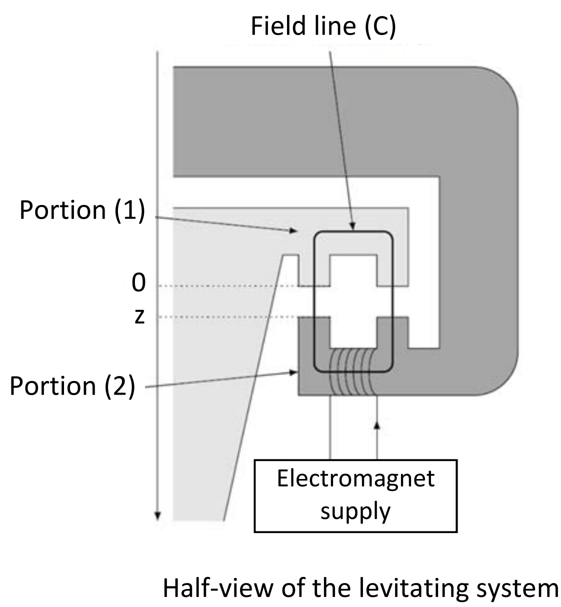


Figure 3: Transrapid's magnetic levitation system detail.

Let us assume that all magnetic field lines are channeled in the magnetic circuit. The air gap width being sufficiently small, we can consider the field lines parallel all along the magnetic circuit. Finally, we consider all field lines to have the same length.

We note:

- $\vec{B}_1$ *is the magnetic field in portion (1),* $\vec{B}_2$ *the magnetic field in portion (2) and* $\vec{B}_a$ *the magnetic field in the air gap,*

- *z is the width of the air gap between the two ferromagnetic portions of the magnetic circuit (the origin O of the downward axis $(O, \vec{u}_z)$ is taken with respect to the fixed rail),*
- *ℓ is the total length of the magnetic field lines (C) which are <u>inside the ferromagnetic portions</u> of the circuit (we will consider $\ell = 20 \cdot z$)*
- *μ_0 and μ are the magnetic permeability of air and of the ferromagnetic material respectively. We note $\mu = \mu_0 . \mu_r$ with $\mu_r = 5000$ and $\mu_0 = 4\pi . 10^{-7}$ F/m.*

It is recalled, moreover, that the reluctance of a magnetic circuit of permeability μ, length l_0 and section S, and surrounded by N loops carrying a current i, is given by $\Re = \frac{l0}{\mu S}$ and verifies the relation : $Ni = \Re \, \phi$.

1) Give Maxwell's equations valid inside the ferromagnetic material.

2) What property verifies the magnetic flux in the magnetic circuit? <u>Establish</u> the relation between B_1, B_2 and B_a.

3) Using the reluctance concept, briefly explain the effect of the air gaps in the magnetic circuit (comparing to the same circuit with no air gap).

4) By properly justifying your reasoning using Ampere's theorem (integral form) express the field B_2 as a function of ℓ, z, N, i_0, μ_0 and μ_r. Simplify the expression obtained by using the preceding questions and the numerical values given in the text.

5) **a)** Show that the self-inductance L of the coil can be written as: $L(z) = \frac{\mu_0 N^2 S}{2z}$.

b) Give the expression of the magnetic energy, E_m, stored in a coil of inductance $L(z)$ through which a current of intensity i_0 is flowing.

c) Show that this energy also corresponds to the magnetic energy stored in the air gaps of the magnetic circuit. (Hint: use the volume density of magnetic energy: $w_m = \frac{1}{2}\frac{B^2}{\mu_0}$).

6) *We will admit that the electromagnetic force exerted on a portion of a magnetic circuit mobile along the direction $\vec{u}_z$, can be written : $\vec{F}_{em} = \frac{\partial E_m}{\partial z}\vec{u}_z$ where E_m is the magnetic energy stored in the circuit.*

a) What is then the literal expression of the electromagnetic force $\vec{F}_{em}$ exerted on the train car ?

b) Calculate the mass m which can be put in levitation in case of air gaps of width z =10 mm and for an electromagnet powered by a current of intensity i_e = 10 A.
Numerical data: N = 1000, S = 0.50 m^2, g = 9.8 $m.s^{-2}$

c) Knowing that the mass of a train car is approximately 190 tons, how many electromagnets are necessary to make the train car levitate? (for a distance train / rail of 10 mm)?

d) **Bonus :** From the expression of the electromagnetic force $\vec{F}_{em}$ established above, draw schematically the evolution of the force with the width of the air gaps and justify why such a system (as described in this exercise) is unstable[3].

[3] *This is why the levitation of the train is regulated in position by an inductive sensor. The system tends to adapt the intensity of the electromagnetic force to a chosen equilibrium position.*