INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON

Second Year FIMI **Year 2020/2021**

Physics Test n° 1

Monday 12th October 2020 **Duration: 1 h 30**

Indicative marking scheme: exercise 1: 6 points, exercise 2: 6 points; exercise 3: 8 points.
Calculator and formula sheet not allowed.

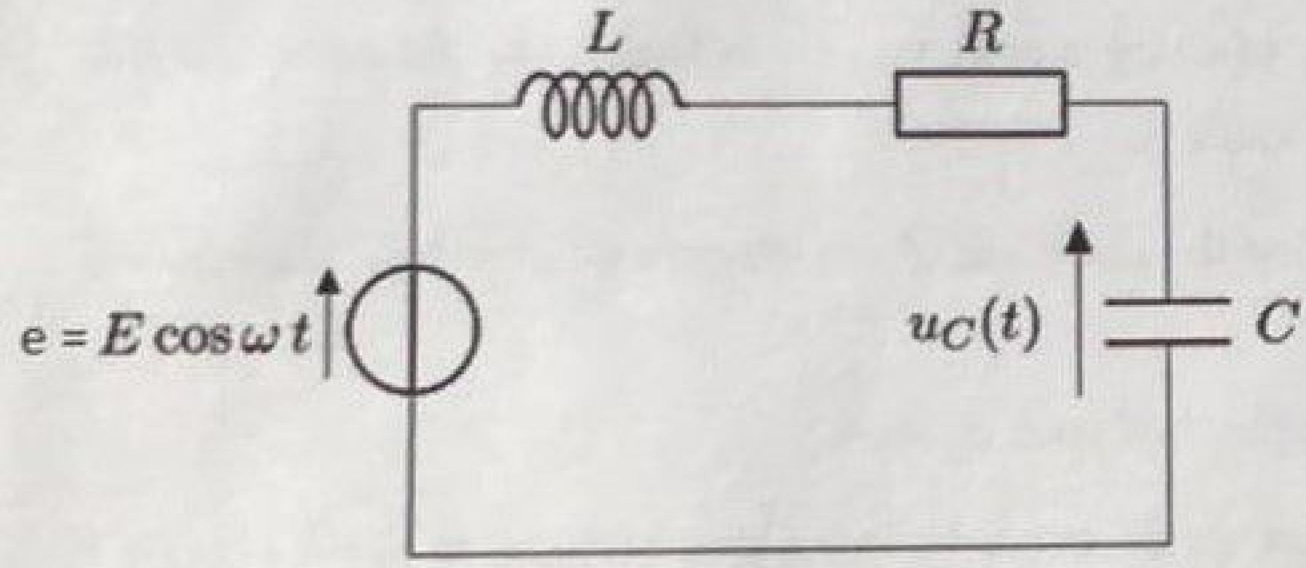
The subject comprises three totally independent exercises.
Any non-homogeneous result will be penalized.

Rotational and divergence in cylindrical coordinates: *for* $\vec{X} = X_r\,\vec{u_r} + X_\theta\,\vec{u_\theta} + X_z\,\vec{u_z}$

$$\overrightarrow{rot}(\vec{X}) = \frac{1}{r}\left(\frac{\partial X_z}{\partial\theta} - \frac{\partial(rX_\theta)}{\partial z}\right)\vec{u_r} + \left(\frac{\partial X_r}{\partial z} - \frac{\partial X_z}{\partial r}\right)\vec{u_\theta} + \frac{1}{r}\left(\frac{\partial(rX_\theta)}{\partial r} - \frac{\partial X_r}{\partial\theta}\right)\vec{u_z}$$

$$div(\vec{X}) = \frac{1}{r}\left(\frac{\partial(rX_r)}{\partial r} + \frac{\partial(X_\theta)}{\partial\theta} + \frac{\partial(rX_z)}{\partial z}\right)$$

1- Electricity:



We consider the circuit represented in figure 1 with R=10 Ω, L=100 mH and C=10 µF.

1/ Determine the literal expression of the complex impedance of the circuit $\underline{Z}$. For which value ω_o of the angular frequency ω does the modulus of $\underline{Z}$ take a minimum value? Calculate the numerical value of ω_o.

2/ Determine the transfer function $\underline{H}(j\omega) = \underline{u_c}(t) / \underline{e}(t)$. What are the limits of the transfer function if the angular frequency is respectively very small or very large?

Express the modulus and the argument of $\underline{H}(j\omega_o)$ and do the numerical application. Sketch succinctly the shape of the graph representing $|\underline{H}(j\omega)|$ as a function of the angular frequency; we will admit that it presents a maximum at an angular frequency very close to ω_o. On this approximative graph indicate the bandwidth of this filter.

3/ We are now interested in the transient state preceding the establishment of a direct current in the same circuit. At $t=0^-$, $e=0$ and the capacitor is initially discharged ($q = 0$). At $t= 0^+$, $e = E_o$ constant value. Establish the differential equation involving q and give the initial conditions applying at $t=0$.

2- Vector Fields:

Consider a straight and infinite electrical conductor (Oz axis) whose circular section has a radius a and through which a current I flows towards increasing z. This current creates a magnetic field $\vec{B} = \frac{\mu_0 I}{2\pi r}\overrightarrow{u_\theta}$ in the air surrounding the wire, with r being a distance to the Oz axis and $\overrightarrow{u_\theta}$ the unitary orthoradial vector in cylindrical coordinates.

We then place a second conductor in the plane xOz, with the same geometry and parallel to the first one, which conducts the same current in the opposite direction. The distance between the axes of the wires is b.

1/ For every point M (x, $y = 0$, z), with $a < x < b\text{-}a$, using Cartesian basis and coordinates, express ~~calculate~~ the magnetic fields $\overrightarrow{B_1}$ et $\overrightarrow{B_2}$ which are created by the first and second conductors respectively. To do that first sketch the set-up and fields in space.

2/ Calculate the flux of $\overrightarrow{B_1} + \overrightarrow{B_2}$ through the surface $a < x < b\text{-}a$, $y = 0$, $0 < z < h$ oriented along $-\overrightarrow{u_y}$.

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3/ Consider $\vec{A} = -\frac{\mu_0 I}{2\pi}\ln\left(\frac{r}{a}\right)\overrightarrow{u_z}$, given in cylindrical coordinates. Verify that $\overrightarrow{B_1} = \overrightarrow{rot}(\vec{A})$. Use this result to calculate the flux of $\overrightarrow{B_1}$ through the surface $a < x < b\text{-}a$, $y = 0$, $0 < z < h$ oriented along $-\overrightarrow{u_y}$.

3- Electrostatics:

In the air, with permittivity ε_0, we place a volume density of charges $\rho(x) = \rho_0 \frac{|x|}{a}$ between two planes: $x = -a$ and $x = +a$ ($\rho(x) = 0$ for $|x| > a$; ρ_0 and a are strictly positive values).

1/ Draw a detailed scheme of the setup and then determine the direction of the electric field and the variables $\vec{E}$ depends on.

2/ Calculate the $\vec{E}$ field everywhere in space using the method of your choice.

3/ Deduce the electrostatic potential V from which it depends for every point in space (x,y,z). We will choose V such that $V(0, 0, 0)=0$.

4/ Provide a graphical representation of the algebraic measure of $\vec{E}(x, 0, 0)$ over a pertinent axis that you shall define.