Alice
Le Bihan

S(AV-)Z

EM : IEFS
(1/3)

58 → 14,32

I) 1) Diffraction occurs when there are interference between a large number of waves.

We need to have : - the same direction of polarization
- coherent waves ($\varphi_1 - \varphi_2 = C^{te}$)
- the same frequency.

Huygen's principle says that upon diffraction, each point M acts like a ^point^ source for a new wave of same frequency and ~~direction of propagation~~ ^spherical^.

2 Given the geometry, we expect rectilinear figures (actually ~~hyperboles~~ that are approximate at infinity as lines). We'll get bright fringes (color of the incident wavelength) and dark fringes). They will be in the ~~x direction~~.

Sketch on
3

2) We have: $\delta_1 = O_1P\sin(i)$
$\delta_2 = O_1P\sin(\theta)$
And $\delta_{1,2} = (\delta_2 + \delta_1)n_0 = O_1P(\sin(\theta)+\sin(i))n_0$

3

/4.5

4) $\underline{da} = A_0 L e^{j(\omega t - k r_2)} dx$

$= A_0 L e^{j(\omega t - k r_1)} e^{-jk(r_2 - r_1)} dx$

3 $= A_0 L e^{j(\omega t - k r_1)} e^{-jk\delta_{12}} dx$

5) $\underline{a} = A_0 L e^{j(\omega t - k r_1)} \int_{\cancel{0}\, -b/2}^{\cancel{X}\, b/2} e^{-jk(\sin(\theta)+\sin(i))x} dx$

$= A_0 L e^{j(\omega t - k r_1)} \frac{\left[e^{-jk(\sin(\theta)+\sin(i))x}\right]_0^X}{-jk(\sin(\theta)+\sin(i))}$

$= A_0 L e^{j(\omega t - k r_1)} \frac{\left(e^{-jk(\sin(\theta)+\sin(i))X} - 1\right)}{-jk(\sin(\theta)+\sin(i))}$

$= A_0 L e^{j(\omega t - k r_1)} \frac{1 - e^{-jk(\sin\theta+\sin i)X}}{jk(\sin(\theta)+\sin i)}$

$= A_0 L e^{j(\omega t - k r_1)} e^{-jk(\sin\theta+\sin i)\frac{X}{2}} \frac{\left(e^{jk(\sin\theta+\sin i)\frac{X}{2}} - e^{-jk(\sin\theta+\sin i)\frac{X}{2}}\right)}{jk(\sin(\theta)+\sin(i))}$

very complicated calculation

$= A_0 L e^{j(\omega t - k r_1)} e^{-jk(\sin\theta+\sin i)\frac{X}{2}} \frac{2j\sin\left(k(\sin\theta+\sin i)\frac{X}{2}\right)}{jk(\sin\theta+\sin i)}$

$= A_0 L e^{j(\omega t - k r_1)} e^{-jk(\sin\theta+\sin i)\frac{X}{2}} X \frac{\sin\left(k(\sin\theta+\sin i)\frac{X}{2}\right)}{\frac{X}{2}k(\sin\theta+\sin i)}$

5.5

$= A_0 \overset{b}{X} L e^{j(\omega t - k r_1)} e^{-jk(\sin\theta+\sin i)\frac{X}{2}} \operatorname{sinc}\left(k(\sin\theta+\sin i)\frac{X}{2}\right)$

please use the variables of the exercise

$\Rightarrow A_{diff}(\theta) = A_0 \overset{b}{X} L e^{-jk(\sin\theta+\sin i)\frac{X}{2}} \operatorname{sinc}\left(k(\sin\theta+\sin i)\frac{X}{2}\right)$ $= b$

6) $I = \langle \underline{a}\cdot\underline{a}^* \rangle = A_0^2 \overset{b^2}{X^2} L^2 \operatorname{sinc}^2\left(k(\sin\theta+\sin i)\frac{\cancel{X}\,b}{2}\right)$

Maximum intensity at $k(\sin\theta - \sin i)\frac{X}{2} = 0$

$\Rightarrow \operatorname{sinc}^2\left(k(\sin\theta - \sin i)\frac{X}{2}\right) = 1$

$\Rightarrow I_0 = A_0 \overset{b^2}{X^2} L^2$

/ ou

4 $\Rightarrow \frac{I}{I_0} = \operatorname{sinc}^2\left(k(\sin(\theta)+\sin i)\frac{X}{2}\right)$

/ ou

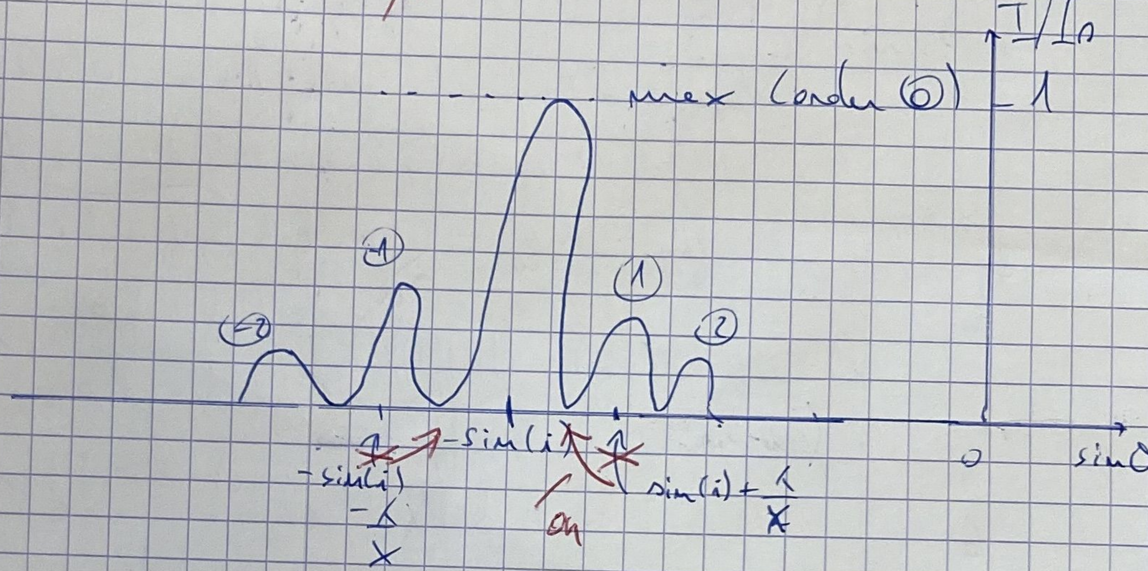
We need $\theta = -i$ for the maximum intensity. Indeed,

1 we want $\sin\theta = -\sin i$ to get $k(\sin\theta - \sin i)\frac{X}{2} = 0$.

/yes

7)

2.5



The order 0 will be when $\delta = 0$

$\Leftrightarrow \sin\theta = \sin i$

look for the zeros of the f°

~~Then order 1 will be when $\delta = \lambda$~~

~~$\Leftrightarrow X(\sin(\theta) + \sin(i)) = \lambda$~~

~~$\Leftrightarrow \sin(\theta) = -\sin(i) + \frac{\lambda}{X}$~~

~~Order 2 will be for $\delta = 2\lambda$ ... etc~~

1 8) We have: $i' = i + \alpha$ and $\theta' = \theta + \alpha$

/yes /ok

9) The expression of the diffracted intensity becomes:

$$\frac{I}{I_0} = \text{sinc}^2\left(k(\sin(\theta+\alpha) + \sin(i+\alpha))\frac{X}{2}\right)$$

1.5 Now we want $\theta' = -(i - \alpha)$

$\Rightarrow \theta' = -(i' - 2\alpha)$

Part II

1) $\Delta = a(\sin i' + \sin \theta')\, n_0$

2) For wave 1 and 3, we'd have:

$$\Delta_{13} = a(\sin i' + \sin\theta') + a\sin(i' + \sin\theta')$$
$$= 2a(\sin(i' + \sin\theta')$$

$$\Rightarrow \text{for } \Delta_m = (m-1)\overbrace{a(\sin(i') + \sin(\theta'))}^{\Delta}$$

2 ok

3) $$a_m(\theta', t) = A_{diff}(\theta')\, e^{j(\omega t - k r_m)}$$
$$= A_{diff}(\theta')\, e^{j(\omega t - k r_1)}\, e^{-jk(r_m - r_1)}$$
$$= A_{diff}(\theta')\, e^{j(\omega t - k r_1)}\, e^{-jk\Delta_m}$$

6 ok

4) $$a_{TOT}(\theta', t) = \sum_1^N A_{diff}(\theta')\, e^{j(\omega t - k r_1)}\, e^{-jk\Delta_m}$$
$$= A_{diff}(\theta')\, e^{j(\omega t - k r_1)} \sum_1^N e^{-jk(m-1)a(\sin(i') + \sin\theta')}$$
$$= A_{diff}(\theta')\, e^{j(\omega t - k r_1)}\, \frac{1 - e^{-jka(\sin i' + \sin\theta')N}}{1 - e^{-jka(\sin i' + \sin\theta')}}$$

Now $1 - e^{-jka(\sin i' + \sin\theta')N}$
$$= e^{-jka(\sin i' + \sin\theta')\frac{N}{2}}\left(e^{jka(\sin i' + \sin\theta')\frac{N}{2}} - e^{-jka(\sin i' + \sin\theta')\frac{N}{2}}\right)$$

$$= e^{-jka(\sin i' + \sin\theta')\frac{N}{2}}\, 2j\sin\left(ka(\sin i' + \sin\theta')\frac{N}{2}\right)$$

And $1 - e^{-jka(\sin i' + \sin\theta')}$
$$= e^{-jka(\sin i' + \sin\theta')\frac{1}{2}}\left(e^{jk\frac{a}{2}(\sin i' + \sin\theta')} - e^{-jk\frac{a}{2}(\sin i' + \sin\theta')}\right)$$
$$= e^{-jk\frac{a}{2}(\sin(i') + \sin(\theta'))}\, 2j\sin\left(k\frac{a}{2}(\sin i' + \sin\theta')\right)$$

Alice Le Bihan

EM TEFS

SCAN72

(2/3)

$$A_{md}\ \frac{e^{-jk\frac{a}{2}(\sin i'+\sin\theta')N}}{e^{-jk\frac{a}{2}(\sin i'+\sin\theta')}}\ \frac{2j\sin\left(k\frac{a}{2}(\sin i'+\sin\theta')\frac{N}{2}\right)}{2j\sin\left(k\frac{a}{2}(\sin i'+\sin\theta')\right)}$$

$$= e^{-jk\frac{a}{2}(\sin i'+\sin\theta')\frac{(N-1)}{2}}\ \frac{\sin\left(k\frac{a}{2}(\sin i'+\sin\theta')\frac{N}{2}\right)}{\sin\left(k\frac{a}{2}(\sin i'+\sin\theta')\right)}$$

$$= e^{-j\frac{2\pi}{\lambda}\cdot\frac{1}{2}\Delta(N-1)}\ \frac{\sin\left(\frac{2\pi}{\lambda}\Delta\frac{N}{2}\right)}{\sin\left(\frac{2\pi}{\lambda}\frac{\Delta}{2}\right)}$$

$$= e^{-j\frac{\pi}{\lambda}\Delta(N-1)}\ \frac{\sin\left(\frac{\pi\Delta}{\lambda}N\right)}{\sin\left(\frac{\pi\Delta}{\lambda}\right)}$$

7

$$\Rightarrow \underline{a}_{TOT}(\theta',t) = A_{diff}(\theta')\ \frac{\sin\left(\frac{\pi}{\lambda}\Delta N\right)}{\sin\left(\frac{\pi\Delta}{\lambda}\right)}\ e^{j(\omega t - k r_1 + \frac{\pi\Delta}{\lambda}(N-1))}$$

Very good

5) $$I_{TOT} = \langle \underline{a}_{TOT}\cdot\underline{a}_{TOT}^{*}\rangle = A^2_{diff}(\theta')\ \frac{\sin^2\left(\frac{\pi}{\lambda}\Delta N\right)}{\sin^2\left(\frac{\pi\Delta}{\lambda}\right)}\cdot\left(e^{-j(\omega t - k r_1 + \pi\frac{\Delta}{\lambda}(N-1))}\cdot e^{j(\omega t - k r_1 + \frac{\pi\Delta}{\lambda}(N-1))}\right)$$

$$= \underbrace{A^2_{diff}(\theta')}_{\text{diffraction enveloppe}}\ \underbrace{\frac{\sin^2\left(\frac{\pi}{\lambda}\Delta N\right)}{\sin^2\left(\frac{\pi\Delta}{\lambda}\right)}}_{\text{interferences of the N rays}}$$

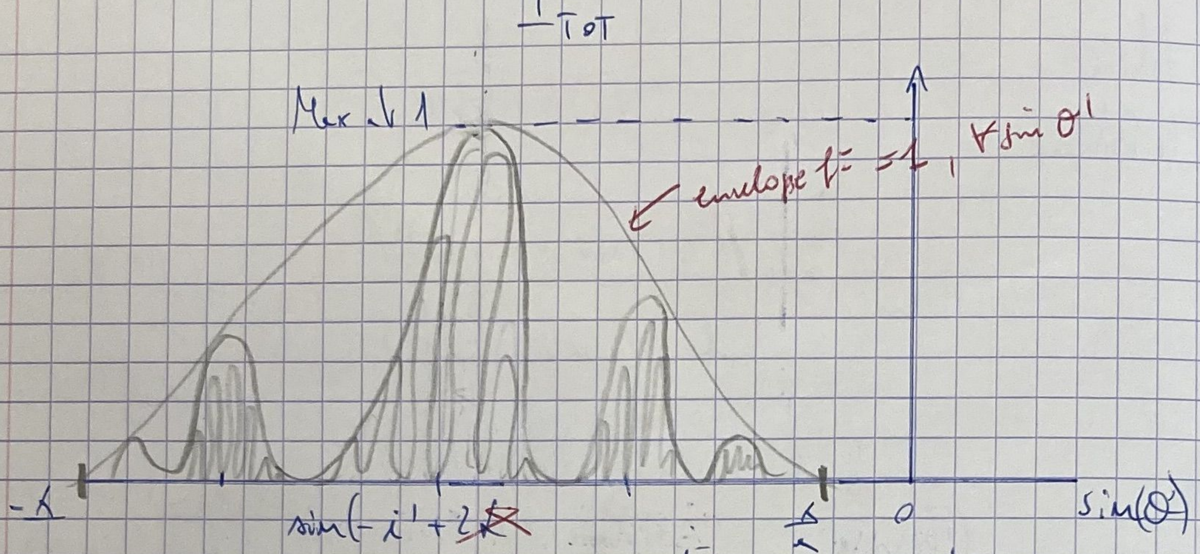
3

interferences of the N rays

diffraction enveloppe

/ok

6) $A_{diff}(\theta') = 1$



We have $A_{diff}\,\theta' = A_0 \times L\, L\, e^{-jk(\sin\theta' + \sin i')\frac{X}{2}} \operatorname{sinc}\left(k(\sin\theta - \sin i)\frac{X}{2}\right)$

To make it independent from θ', we want to increase $\cancel{X}$ ...

Part III

1) We want the maximum of intensity at order p, which means: $\frac{\pi}{\lambda}\,\Delta\cancel{\delta} = p\lambda$

$$\Leftrightarrow a(\sin i' + \sin\theta') = p\lambda$$

$$\Leftrightarrow \sin i' + \sin\theta' = \frac{p\lambda}{a}$$

3

$$\Leftrightarrow \sin\theta' = \frac{p\lambda}{a} - \sin i'$$

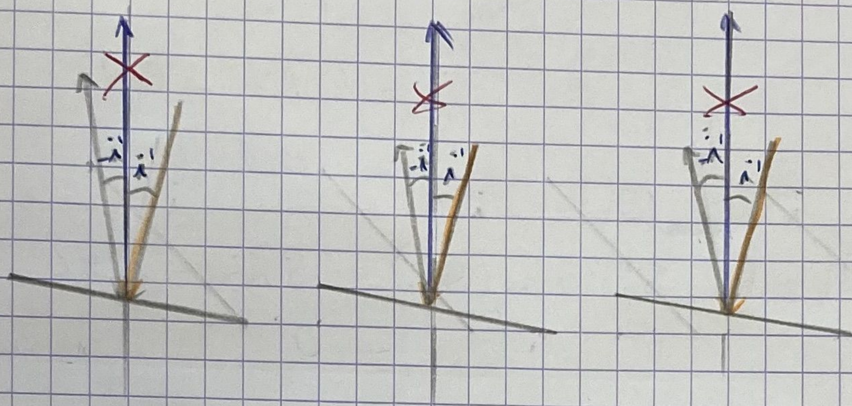
2) Since we have $\theta' = i - \alpha$

we want $\theta' = -(i' - 2\alpha)$

3)

$\longrightarrow$

2



Maximum of order 0 at $\sin(\theta'_p) = -\sin(i')$ /ok

Maximum of A diff : $\theta' = i - \alpha = \alpha - \alpha = \cancel{0}$

4) in this situation, $\theta = i = 0$

$\theta' = \alpha - \alpha = 0$ and $i' = i + \alpha = \alpha$

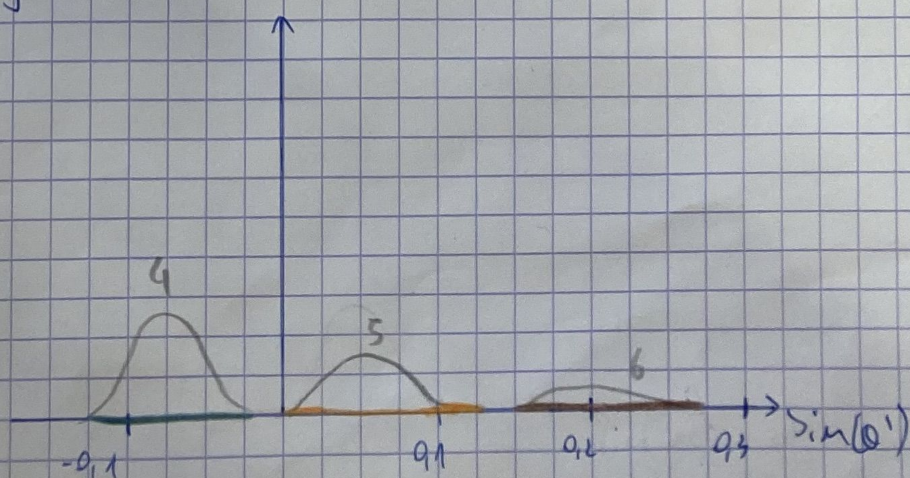
1 We want $0 = 5\frac{\lambda}{a} - \sin(\alpha)$

$\sin(\alpha) = 5\lambda n = 0{,}72$

$\Rightarrow \alpha = \cancel{46{,}0}5°$

5) The grating will separate the wavelengths: we'll get different colors in one order, the bright fringes will take the color of their respective wavelength. 1

6)



At order 4: $\sin_{min}(\theta_4') = p\frac{\lambda}{a} - \sin(\alpha)$

$= 4\lambda n - 0,72$

$= -0,144$

Method ok but results wrong

1

$\sin_{max}(\theta_4') = -0,0544$

At order 5 : $\sin_{min}(\theta_5') = 5\lambda n - 0,72$

$= 0$ and $\sin_{max}(\theta_5') = 0,112$

At order 6: $\sin_{min}(\theta_6') = 0,144$ $\sin_{max}(\theta_6') = 0,2784$

At order 7 the max sin is 0,4448 and at order 8 the min sin is 0,432 => there is superposition starting from order 8.

0.5

7) We want ~~$\frac{N\lambda}{a} > -0,144$~~

Central peak of the diffraction pattern :

???

$$\frac{\pi x}{\lambda} > |-0,144|$$

$$\Leftrightarrow \quad x > \frac{0,144}{\pi}\lambda = 1,63.10^{-8} \text{ m}$$

Alice
Le Bihan

EM TEFS
(3/3)

SCAN

8)

1 9) We have : $R = Np = \frac{\lambda}{d(\lambda lim)}$ $\Rightarrow d(\lambda lim) = \frac{\lambda}{Np}$ /on

1 Working at high order will increase the resolution ($p \nearrow \Rightarrow d(\lambda lim) \searrow$)

/1.5

10) We have: $\Delta_1\lambda = 1,1 \, nm$

$\Delta_2\lambda = 0,6 \, nm$

For the first two wavelength, we need at least $d(\lambda lim) \leq 1,1 \, nm$

$\Leftrightarrow \frac{\lambda}{Np} \leq 1,1 \, nm$

$\Leftrightarrow \frac{\lambda}{1,1 p} \leq N \Rightarrow N \geq 94$

3 For the second, we need $d(\lambda lim) \leq 0,6$

$\Leftrightarrow \frac{\lambda}{Np} \leq 0,6 \Leftrightarrow \frac{\lambda}{0,6 p} \leq N$
$\Rightarrow N \geq 173$

/on