Physics: End of Semester Evaluation

Tuesday 14th June 2022 Time allowed: 3 hours

Indicative marking scheme: Part I: 7, part II: 7, Part III: 6

No documents allowed.
Any non-connected calculator (high school or college types) allowed.
Mobile phones not allowed.

Blazed gratings: Principles and application

A blazed grating is a specific kind of diffraction grating working in reflection. It diffracts light which is ***reflected*** *on its surface due to the presence of ridges (or lines) with a saw-tooth shape (i.e. the edge is like a saw blade). Each ridge (or line) consists of a* ***small plane reflecting surface*** *called a* ***facet*** *in the following. The term "blazed" comes from the fact that it produces intense colors ("blaze" being synonymous to "shine" in English!).*

Since such gratings separate wavelengths very efficiently and with a high resolution, they are often implemented in high-resolution spectrometers. They can be found in several spatial optical instruments like the Hubble telescope for instance.

This test is made up of three parts where we will study successively:

i. *The diffraction due to reflection on an inclined facet (part I),*
ii. *The dispersion of light by a grating made of several facets (part II),*
iii. *The use of such a grating in the context of spectrometry (part III).*

Parts I and II are independent from each other and can be treated independently. On the other hand, part III requires use of the results of the preceding parts.

The air index will be taken equal to 1.

Part I: Diffraction due to reflection on an inclined facet

We consider a ***reflecting facet*** *(small plane reflecting surface). It has* ***a small width b*** *(of the same order of magnitude as the wavelength) and a* ***length L*** *(very large with respect to the wavelength). It is illuminated* ***under oblique incidence*** *by a plane, harmonic, progressive and uniform electromagnetic wave, the incident wave building an* ***incidence angle i*** *with respect to the normal of the facet (Figure 1). In Figure 1, we also define the* ***diffracting angle θ*** *with respect to the normal to the surface. The angles are counted positively clockwise (in Figure 1, i and θ are positive).*

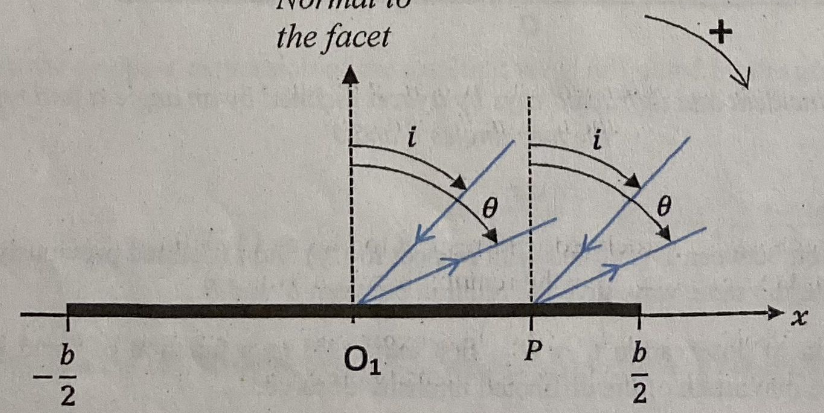

Figure 1: Sketch representing two incident and diffracted rays on a reflecting facet (surface) of small width. ***The angles i and θ are defined with respect to the normal of the facet.***

I.1- Recall under which conditions the diffraction phenomenon is observed, and give a brief outline of Huygens' principal that is the basis of the description of the phenomenon. Given the geometry of the diffracting facet, what diffraction pattern would you expect, and along which axis?

I.2- We are interested in using the sketch given in Annex 1 (to hand in with your manuscript) to visualize the **optical path difference δ** between two rays, one passing through point O_1 and one passing through point P of abscissa x. The incident rays should build the incident angle i whereas the diffracting angle should be θ (source and observation of diffraction are at infinity). Report on the sketch the **angles and distances that are to be considered** to assess the optical path difference δ.

I.3- Express the optical path difference δ as a function of the data of the exercise.

I.4- What is the complex expression of the elementary perturbation $\underline{da}$, of amplitude A_0, diffracted in the direction θ by an elementary source located on the facet at the abscissa x and of width dx and length L? Now, express this elementary perturbation as a function of r_1, the distance covered by the ray passing through O_1 (i.e. the ray diffracted at x=0), and the path difference δ.

I.5- Calculate the complex expression of the resulting wave $\underline{a}$ in the direction of diffraction (and observation) θ. **Deduce from it the expression of its amplitude $A_{diff}(\theta)$.**

I.6- Calculate the expression of the intensity I received by a quadratic detector located at infinity in the direction of observation θ. Deduce then the expression of the normalized intensity I/I_0 where I_0 represents the maximum intensity. For which angle θ is the intensity maximum ($I = I_0$)?

I.7- Sketch on a graph, as a function of $\sin(\theta)$, the evolution of the normalized intensity I/I_0 diffracted by the facet considered. Indicate the important points on the graph.

*Now we **incline the reflecting facet by an angle α** also called "blaze angle" (Figure 2). We denote i' and θ' the new angles of incidence and diffraction respectively which are defined **with respect to the axis Oy** (Figure 2).*

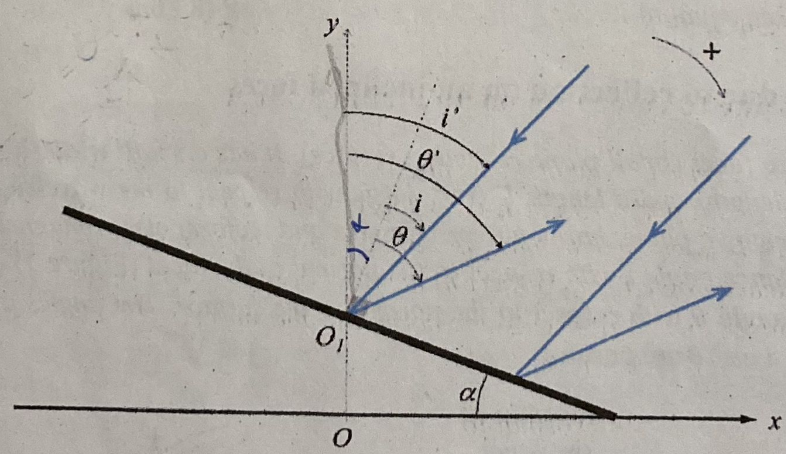

Figure 2: Sketch of two incident and diffracted rays by a facet inclined by an angle α and representation of the new angles i' and θ'

I.8- What is the relation between i' (defined with respect to Oy) and i (defined previously with respect to the normal to the facet)? In the same way, give the relation between θ' and θ.

I.9- In which direction of observation $\theta' = \theta'_1$, first expressed as a function of i and α and then as a function of i' and α, is the maximum of the diffracted intensity observed?

Part II: Dispersion of light by the grating

In this part, the grating is composed of N facets, which are inclined by an angle α with respect to the horizontal plane (see Figure 3).

Now we consider the light diffracted at infinity by each one of the N facets. ***We will assume that each facet behaves like a diffraction source located at its center which emits a wave whose amplitude $A_{diff}(\theta')$ depends on the direction of diffraction θ'. In this part the amplitude $A_{diff}(\theta')$ does not need to be replaced by its actual expression, that can be obtained by expressing θ as a function of θ' in the expression found in Part 1.***

We can consider the phase of the incoming wave as nil at $t = 0$ at the source (located at infinity).

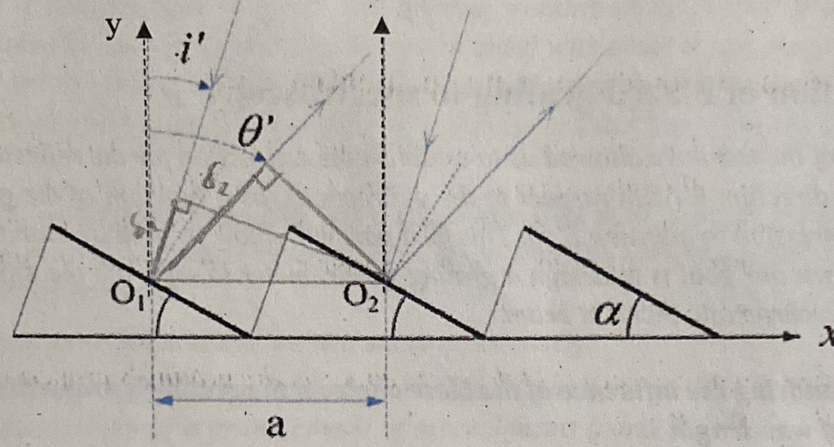

Figure 3: Two incoming rays of light, diffracted by two consecutive facets

II.1- Express the optical path difference $\Delta = \delta_2 - \delta_1$ between the wave diffracted by facet 2 (of centre O_2) in the direction θ' and the wave diffracted by facet 1 (of centre O_1) in the same direction, when the grating is illuminated with a plane wave that arrives with an incidence angle i' (reminder : i' and θ' are defined as in Figure 2, with respect to the axis Oy). To do that complete Figure 3 with the appropriate quantities. Δ will be expressed as a function of a, i' and θ'.

II.2- Deduce the optical path difference Δ_m between the wave diffracted by facet m (with $1 \leq m \leq N$) and the wave diffracted by facet 1.

II.3- Give the complex expression $\underline{a}_m(\theta', t)$ of the wave diffracted by facet m. The expression shall be a function of $A_{diff}(\theta')$, was well as the path length r_1 of the wave diffracted by facet 1, of Δ and m

II.4- Show that the complex expression of the resulting wave diffracted by the grating in the direction θ' is the following:

$$\underline{a}_{tot}(\theta', t) = A_{diff}(\theta') \frac{\sin\left(\frac{\pi N \Delta}{\lambda}\right)}{\sin\left(\frac{\pi \Delta}{\lambda}\right)} e^{j\left(\omega t - k r_1 + \frac{(N-1)\pi\Delta}{\lambda}\right)}$$

We remind you that the sum of the terms of a geometric progression of common ratio q is:

$$1 + q + q^2 + \cdots + q^{N-1} = \frac{1 - q^N}{1 - q}$$

II.5- Show that expression of the total intensity I_{tot} is the following:

$$I_{tot} = A_{diff}^2(\theta').\frac{\sin^2\left(\frac{N\pi\Delta}{\lambda}\right)}{\sin^2\left(\frac{\pi\Delta}{\lambda}\right)}$$

What is the physical significance of the two functions composing the total intensity?

II.6- In the particular case where $A_{diff}(\theta')$ is almost independent from θ' (we will consider $A_{diff}(\theta')$ as being equal to 1 for all θ'), sketch the total intensity as a function of $\sin\theta'$ for $N = 6$ (we note that usually N is much larger). Mark the important values of the function on the sketch. What should be done so as to make $A_{diff}(\theta')$ independent from θ'?

Part III : Application of blazed grating to spectroscopy

The two previous parts of the test have allowed us to establish the expression for the reflected intensity I_{tot} by the blazed grating in a direction θ' (with respect to the y-direction), as a function of the grating parameters and the wavelength (expression in question II.5). This next part will allow us to understand the advantages of using inclined facets when our goal is to design a grating spectrometer to separate the different wavelengths of light making up a polychromatic incident beam.

As a first approach to studying the influence of the blaze angle on the grating's properties, we will consider monochromatic light of wavelength λ.

III.1- Show, using the previously obtained results (eg. from QII.1 and QII.5), that the maximum of order p is found at an angle θ'_p such that : $\sin(\theta'_p) = p\frac{\lambda}{a} - \sin(i')$.

III.2- Recall (e.g. from QI.7 and I.8) for which value of θ' the function $A_{diff}(\theta')$ is maximised, expressed as a function of i' and α.

There is a particularly advantageous configuration for spectroscopy for which the incident wave arrives perpendicularly to the facets (in other words, in the direction given by the normal to the facets). ***We will place ourselves in this configuration for the following questions.***

III.3- Make a diagram of the blazed grating showing the direction of the incident beam, the direction in which $A_{diff}(\theta')$ is maximised, and the direction of the principal order $p = 0$.

III.4- Find the value of the blaze angle α allowing us to superimpose the maximum of the order $p = 5$ with the maximum of $A_{diff}(\theta')$, in the case of a grating of spatial frequency $n = 320$ facets/mm lit by light with wavelength $\lambda = 450$ nm.

We will now consider that the incident light is polychromatic, made up of a continuous domain of wavelengths from 450nm to 520nm.

III.5- Explain how the observed intensity pattern is modified.

Each peak of order p now possesses a certain angular width, within which we observe the dispersion of colours in the wavelength domain [450 nm ; 520 nm]. The blaze angle remains the same as that calculated in question III.4.

III.6- On an axis graduated in $\sin(\theta')$, indicate the angular domains of (or the interval covered by) the orders $p = 4$, $p = 5$, and $p = 6$. Up to which value of p do we not observe a superpositioning of the angular domains of two different orders ?

last page (appendix + a few questions) missing