Written Test n°4

Monday 9th May

Time Allowed : 1h30

***Indicative mark scheme** : exercise 1 : 10 points, exercise 2 : 10 points*

The test is composed of two independant exercises.
Calculator allowed. Mobile phones not allowed. No documents allowed.

Exercise 1 : Oil spills in the Arcachon basin

Following the sinking of the ship 'Grande America' in the bay of Gascogne on 12th March 2019 in the middle of the day, several oil spills were identified on 13th March. In particular in the Arcachon basin, a large quantity of kerosene (of refractive index n) formed a film of low thickness e=0.44µm that covered a large area on the surface of the water, lit by white light from the sun. When this film, or « slick », was seen from an aeroplane, iridescent colours were observed that changed according to the observation angle θ. We will make the assumption that the refractive indices of air, water and kerosene are all real and are respectively $n_0 = 1$, $n_{water} = 1.33$ et $n = 1.448$.

Figure 1 : Arcachon basin and view of oil spill from an aeroplane.

Reminder : *when a plane, uniform wave arrives in near-normal incidence at an interface separating two non-charged dielectric materials (coming from material 1 of index n_1 towards material 2 of index n_2), the amplitude-based reflection and transmission coefficients are :* $r_{1\to 2} = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)$ *et* $t_{1\to 2} = \frac{2n_1}{n_1 + n_2}$.

1) Make a sketch of the situation being studied where you will show the optical path covered by a ray of light when it hits, at oblique incidence with an angle θ, the film of kerosene on the surface of the water.

2) Give the conditions needed to observe interference phenomena. Are they met here ? Justify your answer…

3) *For a given observation angle θ, the angle of refraction in the kerosene is θ_r.* Find (using a clear diagram) the expression of the path length difference δ between the two first rays as a function of the parameters of the problem. We will take into account any contributions from the reflections at interfaces making the hypothesis that such contributions are the same for all angles of incidence.

4) What condition must the path length difference δ satisfy so that the interferences will be constructive ? What is the condition for destructive interference ?

5) *To a first approximation, we can consider that the observed colour of the kerosene film corresponds to wavelengths within the visible spectral region (figure 2) for which the interferences are constructive. The kerosene film is now observed vertically (at normal incidence).*

Give the literal expression of the wavelengths λ_{max} seen with the maximum intensity I_{max} as a function of n, e and p (integer). In the present case, which colour does the kerosene spill appear to be? Justify your answer with a calculation

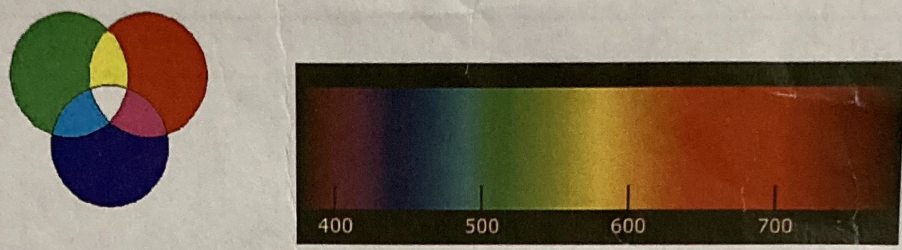

Figure 2 : Additive mixing of colors – Colours and their corresponding wavelength in µm

6) The aeroplane is getting away from the kerosene slick and the observation angle, θ, now reaches 45°. What colour does the kerosene slick now appear to be?
7) The colour of the kerosene spill at the sea's surface doesn't appear to be uniform. The validity of model is brought into doubt. Which explanation(s) could justify this iridescence?

Exercise 2 : Diffraction in a telescope

A refracting telescope is modeled by a slit (F) followed by a thin converging lens (L) of focal distance f' (see annex). The slit (F) has a long height H along $\vec{u}_y$ *and a width L along* $\vec{u}_x$ *. It will be used to describe (in a simplified way) the diffraction at the edges of the telescope.*

We want to observe with this telescope a double star, which means 2 stars close to each other, considered equivalent to 2 independent point sources S_1 *and* S_2*. These two sources are monochromatic and have the same wavelength* $\lambda_0=0.5\mu m$ *in the air of index* $n_0=1$*. The stars are located in the plane of the figure and they are symmetrical with respect to the Oz axis. They are at infinity in two directions making an angle* $i = \varepsilon/2$ *(with* $\varepsilon << 1°$*) with the Oz axis.*

We denote i the angle built by an incident ray with the Oz axis when illuminating the slit. The angle θ *represents the angle built by a ray diffracted by the slit with the Oz axis. We will use the* ***following sign convention****: "the angles of incidence and diffraction of the rays have the same sign if they lie on the same side of the normal to the slit". Moreover the positive orientation of the incidence angles is indicated on the sketch in the annex.*

We are interested in the intensity received in the image focal plane of the lens (L) of the telescope of focal distance f'=50 cm. On the screen we identify the position of the two maxima of intensity which are localized at the points of coordinates : $x' = \pm 10\ \mu m,\ y' = 0$.

1) Describe precisely what we observe in the image focal plane of (L). Justify the presence of the maxima of intensity. What is the angle ε built beween the two rays coming from each of the stars and going to the point O (expressed in radians)?

2) By using the annex joined to this subject, trace the path taken by two parallel rays coming from S_1, passing respectively through the points O and P(x,y) of the slit and reaching point P'. Also trace the path taken by two parallel rays coming from S_2, passing respectively through the points O and P of the slit and reaching point P'.

3) Calculate the optical path difference, denoted δ, between two rays coming from the same source (S_1 or S_2 at your convenience), passing through O and P and received at point P' in such a way that: $\delta = [SPP'] - [SOP']$ *where* $[SPP']$ *is the optical path for the ray passing through P and* $[SOP']$ *is the optical path for the ray passing through O.*

<u>Independent of the source chosen</u>, the angles ε and θ should be present in the calculated expression.