Physics : Written test n°1 (S4)

Monday 18th March Duration : 1h30

Indicative mark scheme*: *exercise 1 out of 6 points, exercise 2 out of 14 points.

The test is composed of two independent exercises.
No documents allowed.
Any non-connected calculator (« collège » / « lycée » types) may be used.
No mobile phones allowed.

Exercise 1: The Saint Pardon bore / a surfer's paradise !

*A **bore** is a wave pulse that travels upstream along a river near its estuary, caused by the interaction between the river's flow and a rising tide (see Figure 1). Despite being dangerous from a navigational point of view, bores are actively sought by surfers who can ride the wave for several kilometers ! There are two well-known bores in France : one in the south-west at the mouth of the Dordogne near Saint-Pardon (best place to see a bore and surfers !), and another, smaller one around the Mont Saint Michel.*

Figure 1 : photograph of a bore near Saint Pardon.

We will consider a bore moving at a velocity c = 20 km · h^{-1} along a straight river, and will define the (Ox) axis following its direction of propagation. At the instant $t_0 = 0$, the profile of the water's surface level y(x,0) is given in Figure 3.a (see Annexe).

1) Draw the profile of the level of the river at time t = 1.0 min, supposing that the wave is not deformed as it propagates: use Figure 3.b in the Annexe **to be handed in**.

2) A surfer is waiting with their surfboard at the point of abscissa x_S = 2.0 km. At what time (hour/minute/second) relative to t_0 will the wave reach them?

3) A fixed sensor placed at the point of abscissa x_d = 1.4 km records the depth of the river as a function of time. Plot the variations $y(x_d,t)$ as a function of t : use Figure 3.c in the Annexe **to be handed in**. Label the three key points (A, B, C).

4) In reality, the wave is attenuated as it propagates. Justify why this might be.

Exercise 2: Electromagnetic wave propagation in salt water

Electromagnetic waves play a major role in the advancement of oceanography. Whether for studying marine ecosystems, mapping the seabed, detecting pollution or monitoring sea ice, the applications of electromagnetic waves are diverse and essential.

Seawater has physical characteristics that depend on salt concentration and temperature. In this exercise, we'll assume that seawater is globally electrically neutral (ρ=0), has a dielectric permittivity $\varepsilon = \frac{9}{4\pi} \cdot 10^{-9}\ F \cdot m^{-1}$, a permeability $\mu_0 = 4\pi \cdot 10^{-7}\ H \cdot m^{-1}$ and a conductivity $\gamma = 4\ S \cdot m^{-1}$.

1) From Maxwell's equations, establish the equation for the propagation of electromagnetic waves in seawater for the electric field $\vec{E}$.

Solutions to this equation are sought in the form of a plane, progressive, harmonic electromagnetic wave (of angular frequency ω), uniform, polarized along (Ox) and propagating in seawater, in the direction of increasing z. The wave is emitted at $z = 0$ and $t = 0$ with a maximum amplitude E_0.

2) Give the complex expression for the electric field $\underline{\vec{E}}(z,t)$ of this wave.
3) Prove that the wave number $\underline{k}$ verifies the equation: $\underline{k}^2 = \mu_0\omega(\varepsilon\omega - j\gamma)$
4) In the case of waves of frequency $f \leq 1\ MHz$, how does the equation verified by $\underline{k}$ simplify?

In the following, we will assume that this condition is satisfied. We introduce $\underline{k} = \mp\frac{1-j}{\delta}$

5) What is the expression of δ as a function of the seawater and wave parameters (among $\mu_0, \varepsilon, \gamma$, ω)?
6) Give the real expression for the electric field $\vec{E}$ (as a function of δ, among other relevant parameters).
7) Derive the expression for the wave's propagation speed (as a function of δ among other relevant parameters).
8) Power calculation :
   a. Find the real expression for the magnetic field $\vec{B}$ (as a function of δ, among other relevant parameters). You should perform this calculation using the complex form of $\vec{E}$, and only take the real part of $\vec{B}$ as the final step to give the required answer.
   b. Find the expression of the Poynting vector $\vec{R}$ (as a function of δ, among other relevant parameters) using the real expressions of $\vec{E}$ and $\vec{B}$.
   c. Calculate the time-averaged power per unit area transported by the wave.

      The following formula may be needed: $\cos A \cos B = \frac{\cos(A-B)+\cos(A+B)}{2}$
   d. At what distance D (called the extinction distance) is this average power per unit area divided by 10?

Numerical application: Electromagnetic waves in certain frequency ranges can penetrate seawater to depths of hundreds of meters, enabling signals to be sent to submarines at the depths where they typically operate. The source emits 3 waves simultaneously, at frequencies of 1 Hz, 1 kHz and 1 MHz.

9) For each frequency, give the value of the wave's propagation speed and its extinction distance D. Conclude on the range of frequencies that allow communication in seawater.

Figure 2: How do submarines communicate with the outside world?

Formulae

Maxwell's complete equations:

$$\overrightarrow{\mathrm{rot}}(\vec{E}) = -\frac{\partial \vec{B}}{\partial \mathrm{t}}, \quad \overrightarrow{\mathrm{rot}}\left(\frac{\vec{B}}{\mu}\right) = \vec{\jmath} + \varepsilon\frac{\partial \vec{E}}{\partial \mathrm{t}}, \qquad div(\varepsilon\vec{E}) = \rho, \qquad \mathrm{div}(\vec{B}) = 0$$

Formula for the curl of a curl of a vector field:

$$\overrightarrow{rot}(\overrightarrow{rot}\vec{E}) = \overrightarrow{grad}(div\vec{E}) - \Delta\vec{E}$$

Instantaneous power per unit surface area carried by an electromagnetic wave:

$$\vec{R} = \vec{E} \wedge \frac{\vec{B}}{\mu}$$

SURNAME : GROUP:

First Name :

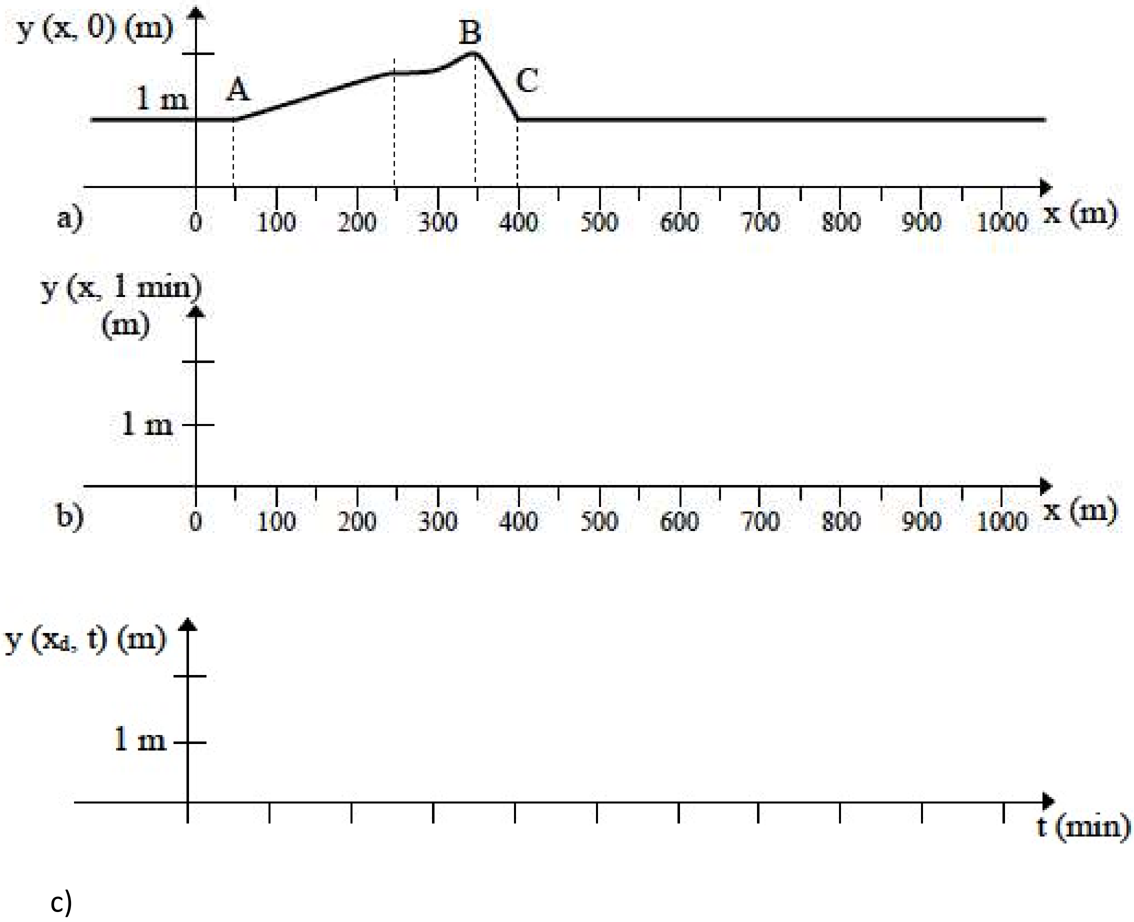

Figure 3. Profiles of the bore.

a) : depth of the water as function of position at the instant t_0 (t = 0).

b) : depth of the water as a function of position at the instant t = 1 min.

c) : depth of the water as a function of time at the position x_d.