INSA INSTITUT NATIONAL DES SCIENCES APPLIQUÉES LYON   FIMI Second Year   Year 2023-2024

Physics: Exam n°2 (S4)

Monday, May the 6th   Duration: 1h

Authorized documents: none;   Calculator not allowed;   Cell phone not authorized

Eyeglass lens treatment

Preliminary remark: this subject is relatively unguided. You should therefore pay particular attention to specifying your reasoning and notations. The quality of your work will be an important criterion in the evaluation of your exam. We also recommend that you take the time to read the presentation carefully.

We model the lens of glasses with its surface treatment (external side) as two media 2 and 3 of respective specific impedance η_2 and η_3 immersed in a medium 1 (air) of impedance η_1. The interface between these different media is flat (see Figure 1). Medium 2 (corresponding to the glass surface treatment) has thickness e. These three media are considered as perfect dielectrics (γ=0), uncharged, with the same permeability μ_0.

Let's consider an undamped, uniform, harmonic plane wave propagating in air (medium 1) and arriving at normal incidence at the interface with medium 2 (surface treatment). The electric field $\overrightarrow{\underline{E}_{1+}}(\vec{r},t)$ associated with this incident wave has amplitude E_0, its wave vector is denoted $\overrightarrow{k_{1+}}(\vec{r},t)$, its phase at the origin is zero. To simplify the problem, the reflection at the interface between medium 3 and air (i.e. at the exit of the glass, see figure 1) is considered negligible.

1) Describe the physical phenomena involved in this stack of three media. In particular, on a diagram of the device, specify the wave vectors of the waves present in each medium, representing them.

   For the notations, the indices 1, 2 or 3 are used for media 1, 2 or 3 respectively, followed by the index **+** for a wave propagating in the increasing x direction, and the index **-** for a wave propagating in the decreasing x direction

It will be assumed hereafter that the electric fields of all waves are always rectilinearly polarized along $\overrightarrow{u_z}$ and that the angular frequency of all electromagnetic waves is not modified by a change of medium.

We can define (in complex notation):

- *a **global** reflection coefficient $\underline{r_g}$, the ratio of the amplitude of the electric field reflected by the GLASS+TREATMENT assembly (field propagating in medium 1 in decreasing x direction), to that of the incident electric field,*
- *a **global** transmission coefficient $\underline{t_g}$, the ratio of the amplitude of the total electric field propagating in increasing x direction in medium 3 (GLASS), to that of the incident electric field,*
- *a complex coefficient $\underline{a}$, the ratio of the amplitude of the total electric field propagating in medium 2 in the direction of increasing x to that of the incident electric field,*
- *and finally a complex coefficient $\underline{b}$, the ratio of the amplitude of the total electric field propagating in the direction of decreasing x in medium 2, to that of the incident electric field.*

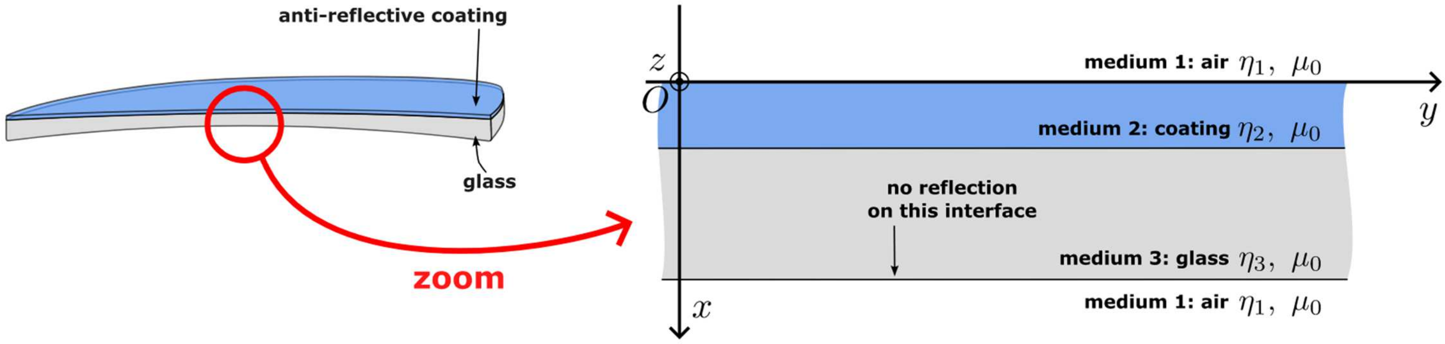

Figure 1: *diagram of glass + treatment system immersed in air*

2) With the help of the diagram made in the previous question, write in complex notation the electric and magnetic fields of the waves present in media 1, 2 and 3, as a function of, among other parameters, E_0, ω, x, V_i (with V_i the velocity in medium i). As previously for the field notations, the indices 1, 2 or 3 are used for media 1, 2 or 3 respectively, followed by the index + for a wave propagating in the increasing x direction, and the index - for a wave propagating in the decreasing x direction (example: $\overrightarrow{\underline{E_{1+}}}(\mathrm{x}, t)$ is the notation for a plane wave of an electric field propagating in medium 1 in the increasing x direction).

3) Determine the 4 equations linking $\underline{r_g}$, $\underline{t_g}$, $\underline{a}$, and $\underline{b}$.

The solution of this system of equations, which is not required here, leads to:

$$\underline{r_g} = \frac{r_{12} + r_{23}e^{-2jk_2e}}{1 + r_{12}r_{23}e^{-2jk_2e}}$$

where, $r_{ij} = \frac{\eta_j - \eta_i}{\eta_i + \eta_j}$ *represents the electric field's reflection coefficient at the interface between a medium of specific impedance* η_i *towards a medium of specific impedance* η_j*. with respect to the incident wave coming from the medium i.*

Note: Recall that the impedance η_i *in a perfect dielectric medium* i*, of permeability µ is related to the propagation velocity* V_i *in this medium by* $\eta_i = \mu\, V_i$*.*

4) Calculate the electric field of the total resulting wave in medium 2 when $\underline{a}$ and $\underline{b}$ are real and $a > b$. Characterize this wave.

We choose a thickness e such that $2k_2e = (2p + 1)\pi$ *(with* $p \in \mathbb{N}$*). In this case the reflection coefficient* $\underline{r_g}$ *becomes:*

$$\underline{r_g} = \frac{\eta_2^2 - \eta_1\eta_3}{\eta_2^2 + \eta_1\eta_3}$$

5) How would you choose the specific impedance of medium 2 so that the treatment is anti-reflective?

6) Compare the optical paths covered by the first two waves reflected by the interface $x = 0$ and $x = e$ respectively (i.e. the first two waves to emerge in medium 1). Why does the thickness e chosen, given that the condition of question (5) is also fulfilled, lead to anti-reflective properties?

7) How does the GLASS+TREATMENT assembly act if $2k_2e = 2p\pi$ (with $p \in \mathbb{N}$) ?