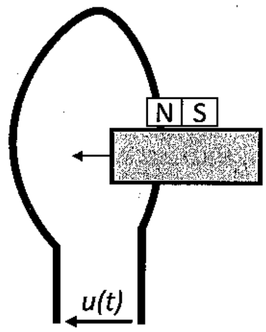
Physics : Test n° 2

Monday January 6, 2025 **Duration : 1h30**

Two-sided A4 handwritten personal form permitted. Calculator in exam mode authorized.
The quality of the writing (thoroughness, explanations, justifications, etc.) will be taken into account.
Indicative marking scheme: Question: 2 pts, Exercise 1 : 7 pts, Exercise 2 : 11 pts

Question : Detection of passing object.

In order to detect the passage of an object through a certain region of space, a permanent magnet is attached to the moving object. A current loop at rest is also used (see figure opposite with the current loop and where the magnet is on top of the moving object in grey).
Using the Faraday-Lenz relation, explain and justify the form of the voltage $u(t)$ observed at the terminals of the coil as the object passes through it (moving from right to left).



Exercise 1 : Electromagnetic weighting scale

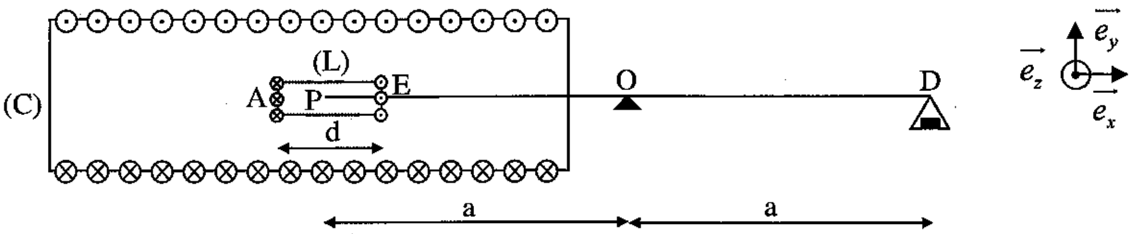


Figure 1

An electromagnetic weighting scale (Figure 1) consists of the following components:
- A solenoid (C), with axis colinear with $\vec{e}_x$, considered of infinite length, comprising n turns per unit length.
- A flat coil (L), with vertical axis colinear with $\vec{e}_y$, square cross-section S, of side d, center P, comprising N turns and placed at the end of the balance arm. Points A and E are defined as being in the middle of segments parallel to $\vec{e}_z$. The coil is rigidly attached to the balance arm.

The same DC current $I > 0$ flows through the solenoid (C) and the flat coil (L) (the currents orientations are shown in Fig. 1). At the other end of the weighting scale's arm, a pan is suspended at point D at distance a from O.
When $I = 0$, the scale is balanced and the beam is horizontal. Adding a mass M to the pan located at point D causes the weighting scale to become unbalanced. The balance can be restored by acting on the current I. The aim of this exercise is to show that it is possible to measure M using this system.
Remember that the magnetic field $\vec{B}$ created by the infinite solenoid (C) is zero outside the solenoid, and is written inside the solenoid: $\vec{B} = \mu_0 nI\ \vec{e}_x$

1- a) A current I flows through the flat coil (L). Draw a diagram showing the force acting on each side of the square coil (L). Calculate the Laplace forces $\vec{F}_A$ and $\vec{F}_E$ acting at points A and E.

b) Calculate the resulting moment of the Laplace forces $\vec{F}_A$ and $\vec{F}_E$ with respect to point O. Determine then the wrench of the magnetic forces (resulting force and moment with respect to point O).

With the balance initially at equilibrium (at $I = 0$), a mass M is placed on pan D.

2- a) Explain precisely how the system consisting of (C) and (L) can be used to rebalance the weighting scale.
b) Determine the expression of M as a function of the current I required to restore balance equilibrium.

N. A.: Calculate numerically M. It is given that $I = 4.4$ A, $a = 20$ cm, d=3cm, surface of the square loop $S = 9$ cm^2, $N = 2000$, $n = 200$ turns/m, $\mu_0 = 4\pi\times10^{-7}$ H/m, g=9.8m.s^{-2}.

Exercise 2 : Non-destructive testing (NDT) technique for detecting defects in metal parts

Preliminary remark: *this exercise is not very guided. In your answers, please detail all the stages of your reasoning to answer the questions asked.*

In order to detect defects such as cracks in a ferromagnetic metal part, an NDT technique involves magnetizing the part to be analysed, so that it can be assimilated to a permanent magnet characterized by a magnetization vector $\vec{M}$.

In this exercise, we'll consider the case where the part to be tested has the shape of a cylindrical sleeve with inner radius a, outer radius b and very long length L (with L >> a and L >> b). It has a uniform magnetization $\vec{M} = M_0\vec{u_z}$ collinear with its unit axis $\vec{u_z}$ for a < r < b (Figure 2.a).

Under these conditions, ***we'll assume that the cylindrical sleeve produces the same magnetic field as that of two (solenoidal) current sheets located one at r = a and the other at r = b, these two sheets being located in a medium of magnetic permeability μ_0 and being crossed by respective surface current densities $\vec{k}_a$ and $\vec{k}_b$ flowing in opposite directions*** *(Figure 2.b).*

Note that the term of current sheet used here means that the current flows over a surface whose thickness is considered to be zero.

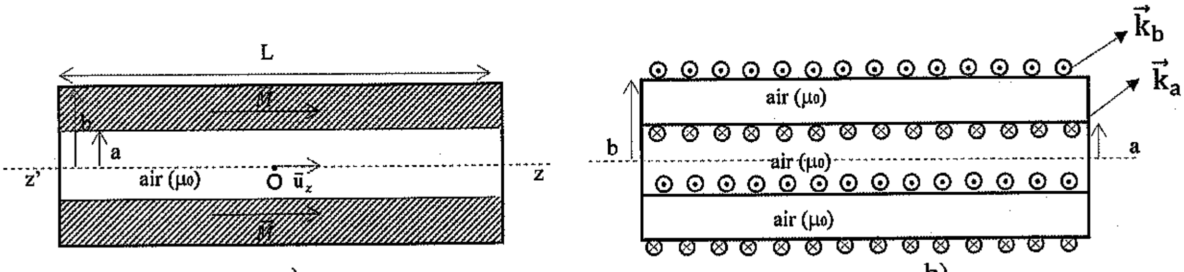


Figure 2 : a) Cylindrical sleeve studied in reality; **b)** Current distribution equivalent to the sleeve (producing the same magnetic field as the sleeve)

1) *The surface current densities associated with the lateral surfaces of the sleeve at r = a and r = b can be calculated by the relationship $\vec{k} = \vec{M} \wedge \vec{N}$ where $\vec{N}$ is a unit vector normal to the surface of the **magnetized medium** and **directed outwards from the medium**.*
   a) make a diagram representing $\vec{N}$ on the surface r=a and r=b respectively. Using the diagram prove that the vectors $\vec{k}_a$ and $\vec{k}_b$ associated with the solenoidal current sheets for r = a and r = b respectively are given by: $\vec{k}_a = -M_0.\vec{u}_\theta$ and $\vec{k}_b = +M_0.\vec{u}_\theta$.
   b) Why is there no current at the two opposite bases (or ends) of the sleeve?
   c) Calculate the currents I_a and I_b associated with the vectors $\vec{k}_a$ and $\vec{k}_b$. What is their sum?

2) *We want to calculate the magnetic field $\vec{B}$ produced by the sleeve at any point in space from the set of equivalent currents shown in Figure 2.b considered in air, knowing that this field is zero at any point such that r > b.* Applying any appropriate point form and/or boundary conditions to the magnetic field $\vec{B}$, give the vector expression of $\vec{B}$ at any point such that r < b.

3) *As a reminder, the magnetic field created by a solenoid is zero outside it.* Show that the magnetic field $\vec{B}$ can also be calculated by applying the superposition principle.

4) *We now consider that the inner surface of the sleeve has a defect similar to a cylindrical groove (filled with air) whose axis coincides with that of the sleeve, with radius c and length 2ℓ much smaller than radius a (Figure 3).*
   To study this configuration, let's consider the model shown in Figure 4, where the cylindrical sleeve with defect is replaced by the superposition of:
   - *Figure 4.a: a cylindrical sleeve without defect of length L, with uniform magnetization $\vec{M} = M_0\vec{u_z}$ generating the magnetic field $\vec{B}$ (calculated previously), and*
   - *Figure 4.b: a cylindrical sleeve of __finite length 2ℓ__, inner radius a, outer radius c, of magnetization $-\vec{M}$, creating the magnetic field $\vec{B'}$.*

Briefly justify this model. Show that the surface current densities, $\overrightarrow{k'_a}$ and $\overrightarrow{k'_c}$, located on the cylinders, of length 2ℓ and of respective radius a and c verify the following relationship: $\overrightarrow{k'_a} = -\overrightarrow{k'_c} = -\vec{k}_a$.

By analogy with the case of the defect-free sleeve of length L (Figure 2), represent on the sleeve of finite length 2ℓ the equivalent current distribution(s)

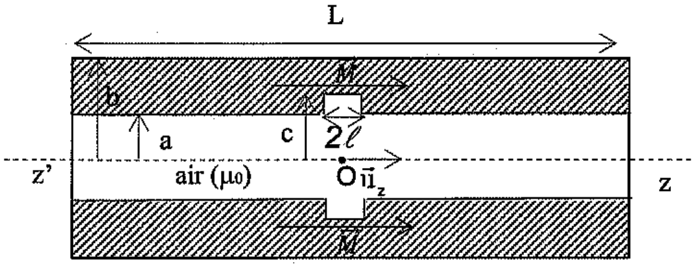
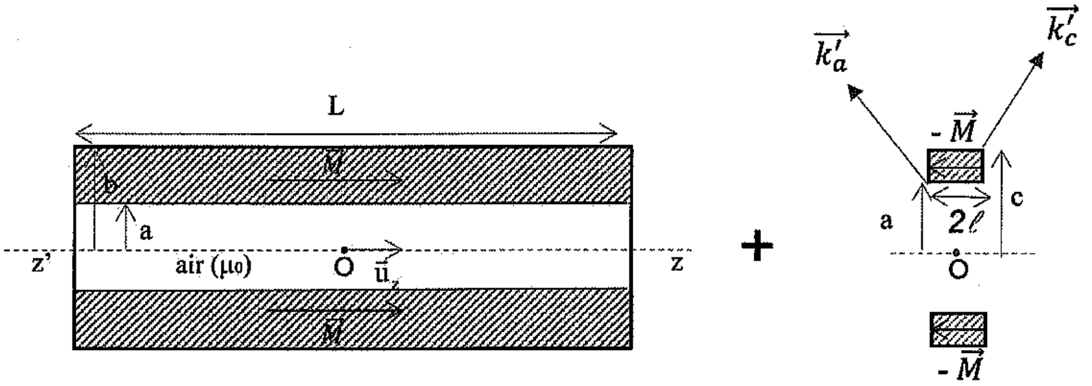


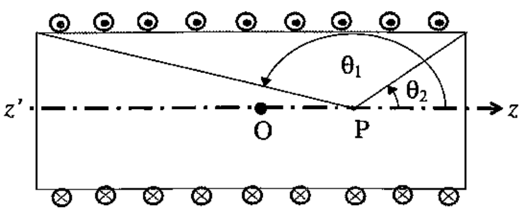
Figure 3 : Cylindrical sleeve with a defect (similar to a cylindrical groove of width 2ℓ and radius c).



a) b)

Figure 4 : Model of the cylindrical sleeve of length L with defect by: **a)** a defect-free sleeve of length L, of uniform magnetization $\vec{M} = M_0\overrightarrow{u_z}$ generating the magnetic field $\vec{B}$ (calculated previously) and **b)** a sleeve of <u>finite length</u> 2ℓ, of magnetization - $\vec{M}$, generating the magnetic field $\overrightarrow{B'}$. *Figure 4.b zooms in on the area where the defect is located.*

The expression of the magnetic field on the axis of a solenoidal current sheet, <u>of finite length</u>, of current density k is given by: $\vec{B}(P) = \frac{\mu_0 k}{2}(\cos\theta_2 - \cos\theta_1)\overrightarrow{u_z}$ where the angles θ_2 and θ_1 are defined in the figure below and considered positive.



5) Using this formula, give the literal expression of the magnetic field $\overrightarrow{B'}$ generated at point O by the current densities $\overrightarrow{k'_a}$ and $\overrightarrow{k'_c}$.
Point O is on the sleeve's z'z axis facing the defect at z = 0 (see figure 4.b).

Considering that $\ell << a$ and $\ell << c$, show that the approximate value of $\overrightarrow{B'}(\boldsymbol{O})$ can be written in the following form: $\overrightarrow{B'}(O) \approx \mu_0 M_0 \ell\left(\frac{1}{a} - \frac{1}{c}\right)\overrightarrow{u_z}$

6) Calculate $\overrightarrow{B'}(O)$ with the following data: $\mu_0 = 4\pi.10^{-7}$ H/m, a = 3 cm, c = 3.2 cm, 2ℓ = 0.6 mm ; $M_0 = 0.8 \times 10^6$ A/m.

7) Based on the above results, explain how to detect a defect (crack) inside a ferromagnetic cylindrical sleeve. What device, whose principle you will explain, could be used to detect and locate the defect?

In cylindrical coordinates :

Scalar field : $f(r,\theta,z)$

Vector field : $\overrightarrow{X} = X_r\ \overrightarrow{u_r} + X_\theta\ \overrightarrow{u_\theta} + X_z\ \overrightarrow{u_z}$

$$\overrightarrow{grad}(f) = \frac{\partial f}{\partial r}\ \overrightarrow{u_r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\ \overrightarrow{u_\theta} + \frac{\partial f}{\partial z}\ \overrightarrow{u_z}$$

$$div\left(\overrightarrow{X}\right) = \frac{1}{r}\ \frac{\partial(rX_r)}{\partial r} + \frac{1}{r}\frac{\partial(X_\theta)}{\partial \theta} + \frac{\partial X_z}{\partial z}$$

$$\overrightarrow{rot}(\overrightarrow{X}) = \left(\frac{1}{r}\frac{\partial X_z}{\partial \theta} - \frac{\partial(X_\theta)}{\partial z}\right)\overrightarrow{u_r} + \left(\frac{\partial X_r}{\partial z} - \frac{\partial X_z}{\partial r}\right)\overrightarrow{u_\theta} + \frac{1}{r}\left(\frac{\partial(rX_\theta)}{\partial r} - \frac{\partial X_r}{\partial \theta}\right)\overrightarrow{u_z}$$