

**Physics - Exam 2**

Friday, May 31, 2024

Duration : 1h30

No document allowed. No mobile phone. Non-programmable calculator and calculator in exam mode allowed. The marks will account not only for the results, but also for the justifications, and the way you analyze the results. The proposed grading scale is only indicative.

**1) The swing (~ 6 points)**

A swing is studied as shown in figure 1. For simplicity, it will be modeled as follows : a point mass (modeling the seat) is connected to a fixed horizontal axis by a single rigid rod of negligible mass.

- Using an energy theorem, determine the minimum speed that must be given to the swing (when it is at its lowest point) for it to make a complete turn, **neglecting friction**. You will fully justify your reasoning and explain any assumption or approach you will be using.

Give the literal expression of the initial velocity and provide an estimation of its order of magnitude.

- We now consider friction whose action will be modeled a constant moment of frictional forces with respect to the swing's rotation axis. Still using an energy theorem, determine the initial speed the swing must be given for it to make a full-turn? We only ask here to determine the literal expression.



FIGURE 1 -

**2) A 2-spring oscillator (~ 6 points)**

An oscillator consists of two identical springs - stiffness constant  $k$ , no-load length  $\ell_0$ . They are attached vertically onto fixed points  $O$  and  $O'$  respectively, separated by a distance  $L = 2\ell_0$ . A mass  $m$  is attached to both springs (Fig. 2) and can only move vertically.

- Determine the equilibrium position  $z_e$  of the mass  $m$ .
- The mass  $m$  is pulled downwards by a distance  $d$  from its equilibrium position. At time  $t = 0$ , it is released without initial velocity. Friction is neglected. Show that the equation of motion of the mass is a differential equation of the form :

$$\ddot{z} + \omega_0^2 z = \omega_0^2 z_e$$

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- Deduce the expression of  $z(t)$  as a function of  $\omega_0$  and  $z_e$  using the initial conditions. Describe the motion of the mass and schematically plot the evolution of  $z(t)$ .

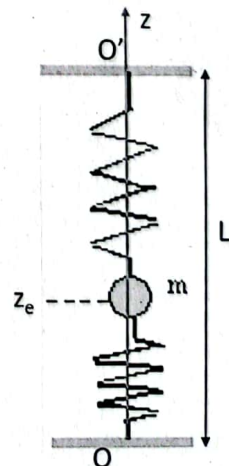


FIGURE 2 - Oscillator at rest

### 3) Receiving a packet (~ 8 points)

An operator has to let a package slide down from point A (top of the slide) to be received at point B (bottom of the slide) by a cart moving along the horizontal floor at a constant speed  $v_c$  (leftwards). When the cart arrives at point  $C_B$ , just below point B, the cart stops for a time interval  $\delta t$  before moving off again (see figure 3). All system elements (package, cart) will be considered as point objects.

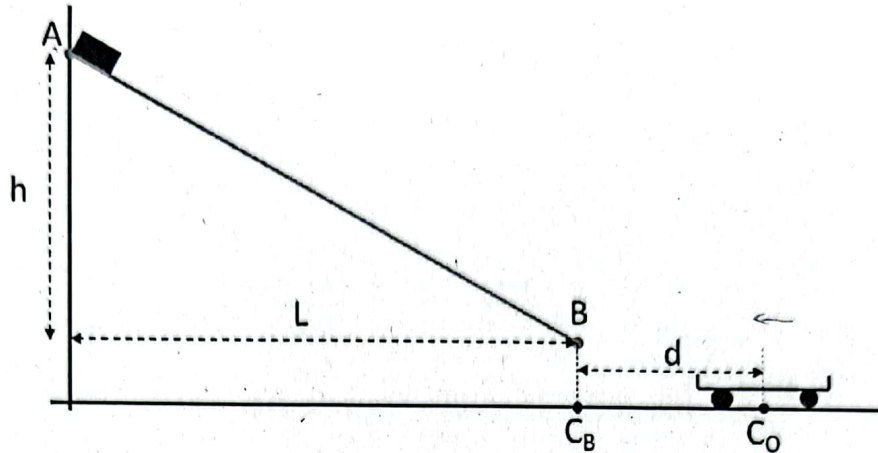


FIGURE 3 - Scheme of the packet delivery system

- 1 - Determine the equation of motion of the packet of mass  $m$  on the slide, knowing that the solid-solid friction coefficient is  $\mu_d$ .
- 2 - Deduce the time taken for the package to reach point B (the package is released without initial speed).
- 3 - The duration of the fall between B and the carriage is neglected in the following. In what time interval must the operator release the package without initial speed so that it can be received by the cart, taking  $t = 0$  s as the moment when the cart leaves point  $C_O$ .

Numerical values :  $g = 9.8 \text{ m s}^{-2}$  ;  $h = L = 4 \text{ m}$  ;  $d = 2 \text{ m}$  ;  $m = 10 \text{ kg}$  ;  $\mu_d = 0.5$  ;  $v_c = 0.5 \text{ m s}^{-1}$  ;  $\delta t = 1 \text{ s}$