Physics exam 3 – Semester 1
February 7th, 2023. Duration: 1h30

No document allowed. No mobile phone. Non-programmable calculator and calculator in exam mode allowed. The proposed grading scale is only indicative.
The marks will account not only for the results, but also for the justifications, and the way you analyze the results. Moreover, any result must be given in its literal form involving only the data given in the text. It is also reminded that the general clarity and cleanness of your paper may also be taken into account.

Exercise 1: Kinematics (~ 8 points)

We study the planar motion of a vehicle on an amusement ride. The vehicle will be considered as a point mass moving on a curved rail in a Galilean frame of reference. We denote $M = (x, y)$ the position of the vehicle in a Cartesian frame $(\vec{u}_x, \vec{u}_y)$ with $\vec{u}_x$ being horizontal, rightwards and $\vec{u}_y$ vertical, upwards.

Over the time interval $[0, 2t_0]$ the motion of M can be fairly described by the following parametric equations:

$$\text{for} \quad 0 \leq t \leq 2t_0 : \quad \begin{cases} x(t) &= \alpha\, t \\ y(t) &= h + \beta(t - t_0)^2 \end{cases} \tag{1}$$

where α, h and β are positive constants.

1. Find the dimensions of α, h and β.
2. Find the equation of the trajectory $y = f(x)$ and provide a qualitative graphical representation (specify some points of interest).
3. Determine the coordinates of the velocity $\vec{v}$ and acceleration $\vec{a}$ in the Cartesian frame.

In the following, the local frame (Frenet frame) will be denoted $(\vec{T}, \vec{N})$, with $\vec{T}$ (resp. $\vec{N}$) being the tangent (resp. normal) unit vector to the trajectory.

4. Find the coordinates of $\vec{v}$ and acceleration $\vec{a}$ in the local frame.
5. What is the radius of curvature of the trajectory (R_c) worth at $t = t_0$?

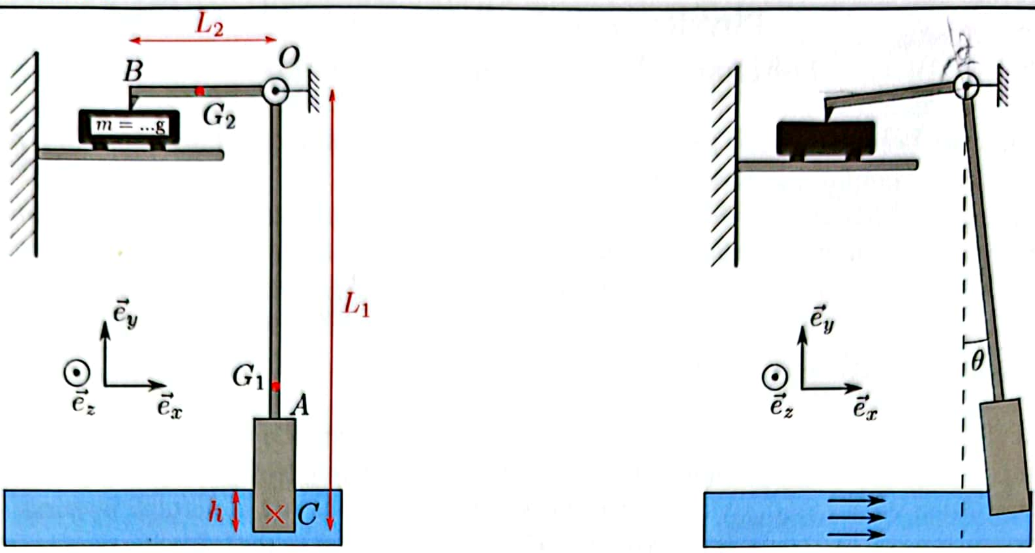
Exercise 2: Flow force measure (~ 12 points)

In a recent study in laboratory LMFA[1], the setup shown in figure 1a was used for measuring the force due to a water flow on an obstacle. The setup consists in a rigid system composed of an obstacle (large grey rectangle) linked to an aluminum bar OA. OA is attached to second aluminum bar OB making a right angle at O. The vertical part {bar OA and obstacle} has a total length L_1, a total mass M_1, with a center of mass G_1 so that $OG_1 = 2/3L_1$. The horizontal bar OB is homogeneous, and has a mass M_2 and length L_2, with a gravity center G_2 situated in its middle point. The system {bars OA and OB and obstacle} can freely rotate without friction around axis Oz. The obstacle is partly immersed in water (depth h). Finally, the extremity of the horizontal bar OB is in point contact with a weighing scale, the latter standing on an immobile horizontal plane.

In the absence of water flow, the system is at equilibrium as sketched in figure 1a, and the scale displays a value m_0. In the presence of a flow, the fluid exerts a force $\vec{F}_D$ on the immersed part, oriented positively along $\vec{e}_x$. Under the influence of $\vec{F}_D$ the system rotates of a small angle θ and reaches a new equilibrium position; the scale then displays a new value denoted m.

Throughout this exercise we will work under the following assumptions:

[1] Laboratoire de Mécanique des Fluides et d'Acoustique, campus LyonTech La Doua



(a) Measurement system at rest (no water flow) (b) Tilted system at equilibrium under flow.

- Since the angle θ is small, $\vec{F}_D$ always remains horizontal (along $\vec{e}_x$) acting as a point force on C. C is located on along (OA) at mid-depth ($h/2$ above the bottom end of the obstacle). The immersion depth remains h whatever θ.
- The reaction force exerted by the scale on OB will be assumed to be vertical upwards (along $\vec{e}_y$) and directly exerted on B (point contact).

System parameters: L_1, M_1, L_2, M_2, h, g (gravitational constant).
Numerical values: $g = 9.81$ m.s^{-2}, $L_1 = 90$ cm, $L_2 = 20$ cm, $M_1 = 800$ g, $M_2 = 200$ g.

1. Explain briefly (*i.e.* without equation) how this system can be used to measure the fluid force on the obstacle.
2. Find the link between the reaction force exerted by the scale on the system and the displayed mass value m.
3. In the absence of water flow, find m_0 in terms of relevant system parameters
4. The flow is now turned on. Show that the magnitude of the fluid force F_D can be written as

$$F_D = \frac{a}{OC}(m - m_0) + b\,\tan\theta \quad (1)$$

   where a and b are functions of the system parameters only, and whose expressions you will provide.
5. Knowing the scale plate vertical displacement is restricted to 1 mm, what is the maximum rotation angle θ_{max}? Show that the maximum relative error made if we neglect the rotation angle is

$$\frac{F_D(\theta = \theta_{max}) - F_D(\theta = 0)}{F_D(\theta = 0)} = \frac{2}{3}\,\frac{M_1}{m - m_0}\,\frac{L_1}{L_2}\tan\theta_{max}$$

   Can we reasonably neglect the effect of the rotation angle for a typical measure $m - m_0 = 400$ g?

In the following, we neglect the rotation angle and we consider F_D to be given by its expression for $\theta = 0$.

The point of application of the fluid force is actually not perfectly known, as it results from a complex distribution of pressure and friction forces on the surface. We thus consider that the application point C can be anywhere between the bottom surface of the obstacle and the water surface (but still along line (OA)).

6. Accounting for this uncertainty on the application point, what is the uncertainty on F_D?
7. How long should be L_1 for the relative uncertainty on F_D to be smaller than 5%, for a water depth $h = 15$ cm?