Physics exam 2 – Semester 1
December 22nd, 2023. Duration: 1h30

No document allowed. No mobile phone. Non-programmable calculator and alculator in exam mode allowed. The proposed grading scale is only indicative.
The marks will account not only for the results, but also for the justifications, and the way you analyze the results. Moreover, any result must be given in its literal form involving only the data given in the text. It is also reminded that the general clarity and cleanness of your paper may also be taken into account.

Exercise 1: Transient regime (~ 10 points)

Consider the electrical circuit depicted in figure 1. The circuit is composed of 3 resistors R_1, R_2, R_3, a coil (inductance L), an ideal voltage source (e.m.f. E) and a switch K.

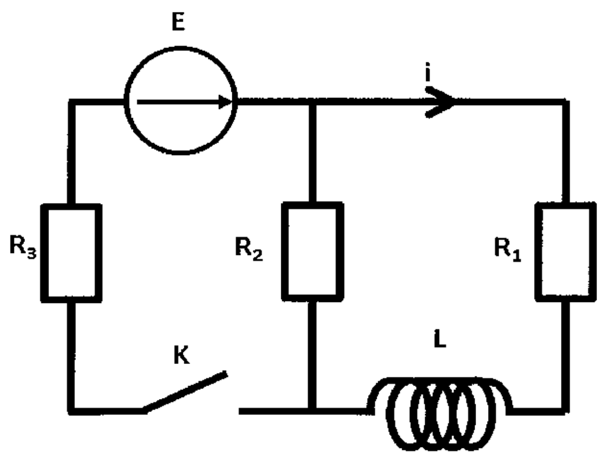

Figure 1: Scheme of the electrical circuit.

1. Determine the steady-state current crossing the coil when K is closed.
   This current will be denoted I_0.
2. We now consider that the switch has remained open for a very long time with respect to the circuit's time constant. At time $t = 0$, the switch K is closed.
   (a) Establish the differential equation governing $i(t)$.
   You may either use circuit transformation or Kirchhoff's circuit laws to answer this question.
   (b) Solve the differential equation using $\tau = \frac{L(R_2+R_3)}{R_1R_2+R_2R_3+R_1R_3}$.
   (c) Make a qualitative plot of $i(t)$ on a graph adding any detail you may judge useful.
   (d) BONUS: Compare the value of $i(t)$ at $t = +\infty$ - denoted I - to I_0 (question 1). Comment.
3. The coil is assumed to be in a DC state (steady-state regime) with a current I running through it. The switch K is then suddenly opened. To simplify the problem, we will assume that K is opened at $t = 0$. The current crossing the coil will be denoted $i'(t)$ from hereon.
   (a) Establish the differential equation governing $i'(t)$.
   (b) Solve the differential equation using $\tau' = \frac{L}{R_1+R_2}$.
   (c) Make a qualitative plot of $i'(t)$.
   (d) Determine the energy dissipated in R_1 and R_2 in the time lapse ranging between the opening the switch and the new steady-state regime.
   (e) What is the origin of this energy? Check the energy balance in the circuit.

Exercise 2: Statics (~ 10 points)

An homogeneous metal rod OA of mass m and length $L = OA$ is suspended from a horizontal wall at point O by a pivot link allowing only rotational movement in the (xOy) plane (Fig. 2-a). The rod is kept in equilibrium by a spring attached between point A and a vertical wall at point B. The distance from O to the vertical wall is L. We denote θ the angle making the rod with the vertical axis.
The spring has the following characteristics: negligible mass, stiffness k and length at rest $\ell_0 \leq L$. Throughout the exercise we will assume that the spring remains horizontal whatever the position of the rod.

Numerical values: L=30 cm, m=250 g, k=20 N m^{-1}, ℓ_0=15 cm and g =9.81 m s^{-2}.

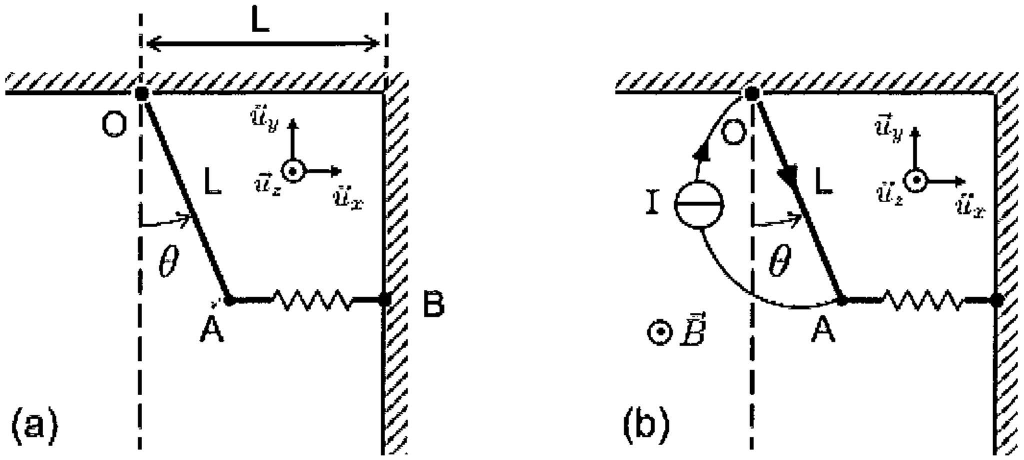

Figure 2: Rod and spring under equilibrium: (a) configuration 1 and (b) configuration 2.

Part 1 - Study of the static equilibrium (~ 5 points)

1. Make a scheme representing all the forces acting on the rod OA and state their characteristics.
2. Show that the equilibrium condition writes: $\alpha \sin(\theta) + \beta \tan(\theta) = \gamma$ with α, β, γ to be expressed in terms of m, L, k and ℓ_0.
3. Assuming θ is small find the expression of the angle at equilibrium θ_{eq} and give its numerical value.
4. Comment the value of θ_{eq} for $\ell_0 = L$ then for $k = 0$.
5. Determine the components of the reaction of the support at O.

Part 2 - Rod crossed by a current I immersed in a $\vec{B}$ field (~ 5 points)

As depicted in Figure 2-b we put an ideal current source in parallel to the rod such that a current $I > 0$ runs from O to A. The rod is then immersed in a uniform magnetic field $\vec{B} = B_0 \vec{u}_z$. Under the action of the magnetic field, the rod gets closer to the vertical axis.
In the following we neglect any influence of the magnetic field on the spring.

1. Give an explanation of the observed phenomenon. Find the expression of the additional force acting on the rod.
2. Determine the magnitude of the magnetic field B_0 allowing the rod to be vertical at equilibrium ($\theta = 0$). Give its numerical value for I=1 A.
3. Comment the value of B_0 for $\ell_0 = L$.